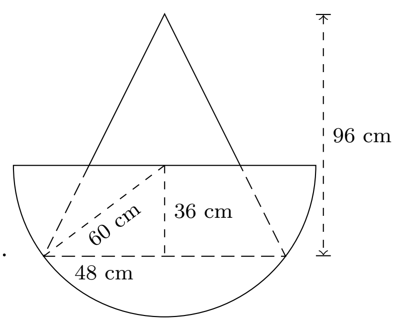
Solution	Marks	Remarks
1. $\frac{m^{-12}n^{8}}{n^{3}} = m^{-12}n^{8-3}$	1M	for $\frac{a^p}{a^q} = a^{p-q}$
$= \frac{n^5}{m^{12}}$	1M+1A	1M for $a^{-p} = \frac{1}{a^p}$
	........(3)	
2. $\frac{3a+b}{8} = b-1$		
$3a+b = 8(b-1)$	1M	for $8(b-1)$
$3a+b = 8b-8$		
$3a = 8b-8-b$	1M	for putting a on one side
$a = \frac{7b-8}{3}$	1A	or equivalent
	........(3)	
3. (a) $x^2-6xy+9y^2$		
$=(x-3y)^2$	1A	or equivalent
(b) $x^2-6xy+9y^2+7x-21y$		
$=(x-3y)^2+7x-21y$	1M	for using (a)
$=(x-3y)^2+7(x-3y)$		
$=(x-3y)(x-3y+7)$	1A	or equivalent
	........(3)	
4. (a) The daily wage of Ada		
$=480(1+20\%)$	1M	
$=\$576$	1A	u−1 for missing unit
(b) The daily wage of Christine		
$=\frac{480}{1-20\%}$	1M	
$=\$600$		
Thus, Christine has the highest daily wage.	1A	f.t.
	........(4)	
5. Let x and y be the number of male and female security guards in each zone respectively.		pp−1 for undefined symbols
$\begin{cases} 6(x+y) = 132 \\ y-x = 4 \end{cases}$	1A+1A	
So, we have $6(2x+4)=132$.	1M	for getting a linear equation in x or y only
Solving, we have $x=4$.		
Thus, the total number of male security guards is 24.	1A	
Let x be the number of male security guards in each zone.		pp−1 for undefined symbols
$6(x+x+4)=132$	1A+1M+1A	1A for $y=x+4$ + 1M for $6(x+y)=132$
Solving, we have $x=4$.		
Thus, the total number of male security guards is 24.	1A	
	........(4)	

Solution	Marks	Remarks
6. (a) $\dfrac{4x+6}{7} > 2(x-3)$		
$4x+6 > 14x-42$		
$-10x > -48$	1M	for putting x on one side
$10x < 48$		
$x < 4.8$	1A	
For $2x-10 \le 0$, we have $x \le 5$.		
Therefore, the solution of $\dfrac{4x+6}{7} > 2(x-3)$ and $2x-10 \le 0$ is $x < 4.8$.	1A	
(b) The positive integers satisfying both inequalities in (a) are 1, 2, 3 and 4. Thus, the number of possible positive integers is 4.	1A	f.t.
	........(4)	
7. (a) $18.1 - a = 6.8$		
$a = 11.3$	1A	
$b - 12.1 = 3.2$		
$b = 15.3$	1A	
(b) Let c s be the longest running time after training.		
$c = 18.1 - 2.9$		
$= 15.2$	1M	
Since $c < b = 15.3$, all the students whose running time fall between 15.3 and 18.1 must have shorter running time after training. Thus, at least 25% students have improved in their running time.	1A	f.t.
	........(4)	
8. (a) $\triangle AED \sim \triangle BEC$	1A	
$\dfrac{AE}{BE} = \dfrac{DE}{CE}$	1M	for using side ratio
$AE = 8 \cdot \dfrac{15}{20}$		
$= 6$	1A	
(b) $AE^2 + BE^2 = 6^2 + 8^2$		
$= 100$		
$= AB^2$	1M	
Thus, $\triangle AEB$ is a right-angled triangle. $AC \perp BD$.	1A	f.t.
	........(5)	

Solution	Marks	Remarks
9. (a) $\dfrac{(BC+AD)\cdot AB}{2}\cdot DE = 1020$	1M	
$\dfrac{(6+AD)\cdot 12}{2}\cdot 10 = 1020$		
$AD = 11$ cm	1A	u−1 for missing unit
(b) $CD = \sqrt{AB^2+(AD-BC)^2}$	1M	for using Pythagoras' theorem
$= \sqrt{12^2+(11-6)^2}$		
$= \sqrt{169}$		
$= 13$ cm	1A	
The total surface area of prism $ABCDEFGH$		
$=2 \times$ Area of $ABCD + DE \times (AB+BC+CD+AD)$		
$=2\cdot\dfrac{(6+11)\cdot 12}{2} + 10\cdot(12+6+13+11)$		
$=624$ cm^2	1A	u−1 for missing unit
	........(5)	
10. (a) mean $= 18$	1A	
median $= 16$	1A	
	........(2)	
(b) (i) The mean number of hours of the 24 questionaires		
$=\dfrac{18\times 20+18\times 4}{24}$		
$=18$	1A	
(ii) Let a and b be the number of hours of the other 2 questionaires of the last 4 questionaires.		pp−1 for undefined symbol
$\dfrac{a+b+19+20}{4} = 18$		
$a+b = 33$	1M	
If the median number of hours of the 24 questionaires is the same as the median in (a), both a and b must be less than or equal to 16, i.e. $a+b \leq 32$.	1M	
Thus, it is impossible the new median to be the same as the median in (a).	1A	f.t.
	........(4)	
11. (a) Let $C = a + bA$, where a and b are non-zero constants.	1A	
So, we have $a+2b = 62$ and $a+6b = 74$.	1M	for either substitution
Solving, we have $a = 56$ and $b = 3$.	1A	for both correct
The required cost		
$=56+3(13)$		
$=\$ 95$	1A	u−1 for missing unit
	........(4)	
(b) The surface area of the larger can		
$=13\times(\sqrt[3]{8})^2$	1M	for using similarity
$=52$ m^2		
The required cost		
$=56+3(52)$		
$=\$ 212$	1A	u−1 for missing unit
	........(2)	

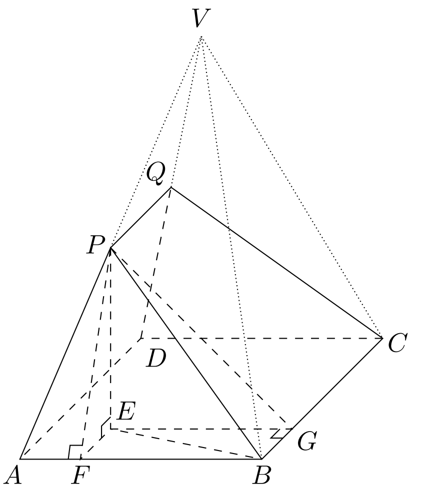
Solution	Marks	Remarks
12. (a) The volume of the cone		
$=\frac{1}{3}\pi r^2 h$	1M	for $V=\frac{1}{3}\pi r^2 h$
$=\frac{1}{3}\pi \cdot 0.48^2 \cdot 0.96$		
$=0.073728\pi$ m^3	1A	u−1 for missing unit 0.0737π
	........(2)	
(b) (i) The volume of the milk		
$=\frac{1}{2}\cdot\frac{4}{3}\pi r^3$	1M	for $V=\frac{4}{3}\pi r^3$
$=\frac{2}{3}\pi \cdot 0.6^3$		
$=0.144\pi$ m^3	1A	u−1 for missing unit
(ii) The height of the portion of the cone in the milk		
$=\sqrt{0.6^2-0.48^2}$	1M	for using Pythagoras' theorem
$=0.36$ m		
The volume of the portion of the cone in the milk		
$=(0.073728\pi)\left[1-\left(\frac{0.96-0.36}{0.96}\right)^2\right]$	1M	for using similarity
$=0.044928\pi$ m^3		
The volume of the remaining milk in the container		
$=0.144\pi-0.044928\pi$		
$=0.311$ m^3		
Thus, the volume of the remaining milk in the container is larger than 0.3 m^3.	1A	f.t.
	........(5)	
13. (a) Since $x-2$ is a factor of $kx^3-21x^2+24x-4$, we have		
$k(2^3)-21(2^2)+24(2)-4=0$	1M	
$8k=40$		
$k=5$	1A	
	........(2)	
(b) (i) The coordinates of R		
$=(0,15m^2-63m+72)$	1M	
The area of rectangle $OPQR$		
$=(m)(15m^2-63m+72)$		
$=15m^3-63m^2+72m$	1A	
(ii) If the area of rectangle $OPQR$ is 12, m satisfy		
$15m^3-63m^2+72m=12$		
$5m^3-21m^2+24m-4=0$		
$(m-2)(5m^2-11m+2)=0$	1M+1A	1M for $(m-2)(am^2+bm+c)$
$(m-2)(5m-1)(m-2)=0$		
$m=\frac{1}{5}$ or $m=2$ (repeated)		
Thus, there are only 2 possible locations of Q such that the area of rectangle $OPQR$ is 12.	1A	f.t.
	........(5)	

Solution	Marks	Remarks
14. (a) (i) Γ and L are parallel with Γ lies below L by a vertical distance of 1.	1A+1A	1A for parallel + 1A for vertical distance
(ii) The equation of L is		
$\frac{x}{3}+\frac{y}{-1}=1$	1M	for intercept form
$y=\frac{x}{3}-1$		
Thus, the equation of Γ is		
$y=\frac{x}{3}-1-1$	1M	
$x-3y-6=0$	1A	
	........(5)	
(b) (i) The coordinates of Q		
$=(6,0)$		
Subtituting the coordinates of Q into the equation of Γ, we have		
L.S. $=6-3(0)-6$		
$=0$		
$=$ R.S.		
Thus, Q passes through Γ.	1A	
(ii) Since Q passes through Γ, $HQ=KQ=$ radius of C.	1M	
Let F and G be the feet of the perpendiculars from A and B to Γ respectively.		
$AF=BG=$perpendicular distance from L to Γ.	1M	
Hence, we have		
Area of $\triangle AQH$: Area of $\triangle BQK$		
$=\frac{1}{2}HQ\cdot AF:\frac{1}{2}KQ\cdot BG$		
$=1:1$	1A	
	........(4)	
15. (a) The standard deviation after the adjustment		
$=10\times(1+20\%)$		
$=12$	1A	
	........(1)	
(b) Let μ be the mean score before the adjustment. Let x_i be the test score of a student before the adjustment.		
The standard score of that student after the adjustment		
$=\frac{(1.2x_i+5)-(1.2\mu+5)}{12}$	1A	for mean score after adjustment
$=\frac{1.2(x_i-\mu)}{12}$		
$=\frac{x_i-\mu}{10}$		
Thus, the standard score of each student remains unchange.	1A	f.t.
	........(2)	

Solution	Marks	Remarks
16. (a) The required probability		
$=\dfrac{C_4^8(C_1^2)^4}{C_4^{16}}$	1M	for numerator or demoninator
$=\dfrac{8}{13}$	1A	r.t. 0.615
The required probability		
$=\left(\dfrac{14}{15}\right)\left(\dfrac{12}{14}\right)\left(\dfrac{10}{13}\right)$	1M	for $\left(\dfrac{n-2}{n-1}\right)\left(\dfrac{n-4}{n-2}\right)\left(\dfrac{n-6}{n-3}\right)$
$=\dfrac{8}{13}$	1A	r.t. 0.615
	........(2)	
(b) The required probability		
$=1-\dfrac{8}{13}$	1M	for 1− (a)
$=\dfrac{5}{13}$	1A	r.t. 0.385
The required probability		
$=\dfrac{C_2^8+C_3^8(C_1^2)^2}{C_4^{16}}$	1M	for considering 2 cases
$=\dfrac{5}{13}$	1A	r.t. 0.385
The required probability		
$=\dfrac{1}{15}+\left(\dfrac{14}{15}\right)\left(\dfrac{2}{14}\right)+\left(\dfrac{14}{15}\right)\left(\dfrac{12}{14}\right)\left(\dfrac{3}{13}\right)$	1M	for considering 3 cases
$=\dfrac{5}{13}$	1A	r.t. 0.385
	........(2)	

Solution	Marks	Remarks
17. (a) Since C touches the x-axis, the radius of C is 10.	1A	can be absorbed
The equation of C is		
$(x-6)^2+(y-10)^2=10^2$	1A	
	........(2)	
(b) The equation of L is $y=-x+k$.	1M	
Substituting $y=-x+k$ into the equation of C, we have		
$(x-6)^2+(-x+k-10)^2=10^2$		
$2x^2-2(k-4)x+(k-10)^2-64=0$		
Let (x_0,y_0) be the mid-point of AB.		
$x_0=\dfrac{k-4}{2}=\dfrac{k}{2}-2$	1M+1A	1M for using sum of roots
$y_0=-\left(\dfrac{k}{2}-2\right)+k=\dfrac{k}{2}+2$	1A	
Thus, the mid-point of $AB=\left(\dfrac{k}{2}-2,\dfrac{k}{2}+2\right)$.		
The equation of L is $y=-x+k$.	1M	
The line passing through (6, 10) and perpendicular to L has equation		
$\dfrac{y-10}{x-6}=1$	1A	
$y=x+4$		
Substituting $y=-x+k$ into the equation, we have		
$-x+k=x+4$	1M	
$x=\dfrac{k}{2}-2,\quad y=\dfrac{k}{2}+2$	1A+1A	
	........(5)	

Solution	Marks	Remarks
18. (a) By sine formula,		
$\dfrac{PA}{\sin\angle PBA}=\dfrac{AB}{\sin\angle APB}$	1M	
$\dfrac{PA}{\sin 60^\circ}=\dfrac{20}{\sin(180^\circ-72^\circ-60^\circ)}$		
$PA\approx 23.30704256$		
$PA\approx 23.3$	1A	r.t.
	........(2)	
(b) (i) Let E, F, G be the feet of the perpendicular of P to the plane $ABCD$, the line AB and the line BC respectively.		
$\dfrac{AF}{PA}=\cos\angle PAB$		
$AF\approx 7.20227224$		
$\dfrac{AF}{AE}=\cos\angle EAF$		
$AE\approx\dfrac{7.20227224}{\cos 45^\circ}$		
≈ 10.18555108		
$PE=\sqrt{PA^2-AE^2}$	1M	
$\approx\sqrt{23.30704256^2-10.18555108^2}$		
≈ 20.96360613		
$EG=FB$	1M	
$=AB-AF$		
≈ 12.79772776		
$\tan\alpha=\tan\angle PGE$	1M	for identifying the angle
$=\dfrac{PE}{EG}$		
≈ 1.638072518		
$\alpha\approx 58.59703733^\circ$		
$\alpha\approx 58.6^\circ$	1A	
(ii) $\tan\beta=\tan\angle PBE=\dfrac{PE}{EB}$	1M	for identifying the angle
$EB>EG$		
$\dfrac{PE}{EB}<\dfrac{PE}{EG}$		
$\tan\beta<\tan\alpha$		
$\beta<\alpha$	1A	f.t.
	........(6)	



Solution	Marks	Remarks
19. (a) (i) $\begin{cases} A(1)=ab^2=254100 \\ A(2)=ab^{2(2)}=307461 \end{cases}$		
Solving, we have $b=1.1$ and $a=210000$.	1A+1A	
$A(4)=ab^{2(4)}$ $=(210000)(1.21^4)$ $=450,153.6501$		
Thus, the weight of the cargos handled in the 4-th year is 450,153.6501 tonnes.	1A	r.t. 450,000
(ii) The total weight of the cargos handled in n years		
$=\sum_{i=1}^{n} A(i)$	1M	for summing up $A(i)$
$=\dfrac{ab^2(b^{2n}-1)}{b^2-1}$	1M	for $\sum_{i=1}^{n} kr^i = \dfrac{kr(r^n-1)}{r-1}$
$=\dfrac{(210,000)(1.21)(1.21^n-1)}{1.21-1}$		
$=1,210,000(1.21^n-1)$ tonnes	1A	
	........(6)	
(b) (i) $\dfrac{A(n)}{B(m)}=\dfrac{ab^{2n}}{2ab^m}$	1M	for considering the ratio $\dfrac{A(n)}{B(m)}$
$=\dfrac{b^{2n}}{2b^{n-4}}$	1A	for changing m to $n-4$
$=\dfrac{1}{2}1.1^{n+4}$		
≤ 1 for $n \leq \log_{1.1} 2 - 4 \approx 3.27$		
Thus, for the first 3 years, the weight of the cargos handled by Y is larger than that by X.	1A	f.t.
(ii) The total weight of the cargos handled in n years by X and Y		
$=\sum_{i=1}^{n} ab^{2i} + \sum_{i=1}^{n-4} 2ab^i$	1A	for correct upper limits
$=1,210,000(1.21^n-1)+\dfrac{2(210,000)(1.1)(1.1^{n-4}-1)}{1.1-1}$		
$=1,210,000(1.1^{2n})+\dfrac{4,200,000}{1.331}(1.1^n)-5,830,000$		
Let $x=1.1^n$, we have		
$1,210,000x^2+\dfrac{4,200,000}{1.331}x-5,830,000>20,000,000$	1M	for setting up a quadratic equation
$1,610,510x^2+4,200,000x-34,379,730>0$		
$x<-6.104700691$ (rejected since $1.1^n>0$) or $x>3.496831134$.	1A	
Thus, we have		
$1.1^n>3.496831134$ $n>13.13455888$		
The new facility should be installed in the 13-th year.	1A	
	........(7)	

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MATHEMATICS COMPULSORY PART PAPER 2

Question No.	Key	Question No.	Key
1.	C	31.	B
2.	D	32.	C
3.	C	33.	A
4.	B	34.	C
5.	B	35.	A
6.	D	36.	D
7.	C	37.	A
8.	D	38.	C
9.	A	39.	D
10.	D	40.	D
11.	C	41.	C
12.	B	42.	B
13.	D	43.	B
14.	B	44.	D
15.	A	45.	A
16.	B		
17.	B		
18.	A		
19.	C		
20.	C		
21.	D		
22.	A		
23.	D		
24.	A		
25.	C		
26.	A		
27.	A		
28.	B		
29.	B		
30.	D		