

**香港考試及評核局**  
**HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY**

**2 0 1 2 年 香 港 中 學 文 憑 考 試**  
**HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2012**

**數學 延伸部分**  
**單元一（微積分與統計）**

**MATHEMATICS Extended Part**  
**Module 1 (Calculus and Statistics)**

評 卷 參 考

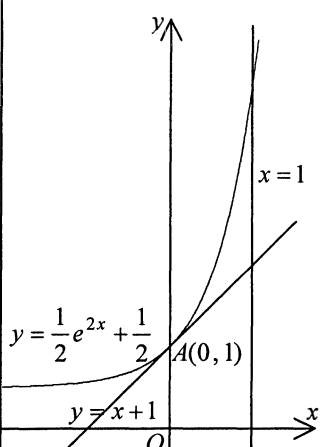
MARKING SCHEME

本評卷參考乃香港考試及評核局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

Solution	Marks	Remarks																		
1. (a) $(1+3x)^n = 1 + C_1^n(3x) + C_2^n(3x)^2 + \dots$ $= 1 + 3nx + \frac{9n(n-1)}{2}x^2 + \dots$	1A																			
(b) $e^{-2x}(1+3x)^n = \left[ 1 + (-2x) + \frac{(-2x)^2}{2!} + \dots \right] \left[ 1 + 3nx + \frac{9n(n-1)}{2}x^2 + \dots \right]$ $= (1 - 2x + 2x^2 + \dots) \left[ 1 + 3nx + \frac{9n(n-1)}{2}x^2 + \dots \right]$ $\therefore 1 \cdot \frac{9n(n-1)}{2} + (-2)(3n) + 2 \cdot 1 = 62$ $9n^2 - 21n - 120 = 0$ $n = 5$ or $\frac{-8}{3}$ (rejected)	1A 1M 1A	For $1 + (-2x) + \frac{(-2x)^2}{2!} + \dots$																		
	(4)																			
2. Let $u = 4t + 1$ . $du = 4dt$ When $t = 0$ , $u = 1$ ; when $t = 2$ , $u = 9$ . The change in the value of the flat $= \int_0^2 \frac{t}{\sqrt{4t+1}} dt$ $= \int_1^9 \frac{1}{\sqrt{u}} \cdot \frac{u-1}{4} \frac{du}{4}$ $= \frac{1}{16} \int_1^9 \left( \sqrt{u} - \frac{1}{\sqrt{u}} \right) du$ $= \frac{1}{16} \left[ \frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right]_1^9$ $= \frac{5}{6}$ Hence the percentage change $= \frac{\frac{5}{6}}{3} \times 100\%$ $= 27\frac{7}{9}\%$	1M 1M 1A 1A 1A	For $\frac{1}{16} \left[ \frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right]$  OR 27.7778%																		
	(5)																			
3. (a) $P = ae^{\frac{kt}{40}} - 5$ $\ln(P+5) = \frac{k}{40}t + \ln a$	1A																			
(b) <table border="1" style="margin-left: 20px;"> <tr> <td><math>t</math></td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> </tr> <tr> <td><math>P</math></td> <td>2.36</td> <td>2.81</td> <td>3.23</td> <td>3.55</td> <td>4.01</td> </tr> <tr> <td><math>\ln(P+5)</math></td> <td>2.00</td> <td>2.06</td> <td>2.11</td> <td>2.15</td> <td>2.20</td> </tr> </table>	$t$	2	4	6	8	10	$P$	2.36	2.81	3.23	3.55	4.01	$\ln(P+5)$	2.00	2.06	2.11	2.15	2.20	1M	
$t$	2	4	6	8	10															
$P$	2.36	2.81	3.23	3.55	4.01															
$\ln(P+5)$	2.00	2.06	2.11	2.15	2.20															
From the graph on the next page, $\ln a \approx 1.96$ $a \approx 7$ $\frac{k}{40} \approx \frac{2.21 - 1.96}{10 - 0}$ $k \approx 1$	1M 1A	<div style="border: 1px solid black; padding: 5px; display: inline-block;">             Either one           </div>																		
	1A	For both $a$ and $k$																		

Solution	Marks	Remarks
	1A	
	(5)	
<p>4. (a) <math>y = \sqrt[3]{\frac{3x-1}{x-2}}</math></p> $\ln y = \frac{1}{3} \ln(3x-1) - \frac{1}{3} \ln(x-2)$ $\therefore \frac{1}{y} \frac{dy}{dx} = \frac{1}{3x-1} - \frac{1}{3(x-2)}$ <p>(b) By (a), <math>\frac{dy}{dx} = \left[ \frac{1}{3x-1} - \frac{1}{3(x-2)} \right] \sqrt[3]{\frac{3x-1}{x-2}}</math></p> $\frac{d^2y}{dx^2} = \frac{d}{dx} \left[ \frac{1}{3x-1} - \frac{1}{3(x-2)} \right] \sqrt[3]{\frac{3x-1}{x-2}} + \left[ \frac{1}{3x-1} - \frac{1}{3(x-2)} \right] \frac{d}{dx} \left( \sqrt[3]{\frac{3x-1}{x-2}} \right)$ $= \left[ \frac{-3}{(3x-1)^2} + \frac{1}{3(x-2)^2} \right] \sqrt[3]{\frac{3x-1}{x-2}} + \left[ \frac{1}{3x-1} - \frac{1}{3(x-2)} \right]^2 \sqrt[3]{\frac{3x-1}{x-2}} \quad \text{by (a)}$ <p>When <math>x=3</math>, <math>\frac{d^2y}{dx^2} = \left\{ \frac{-3}{(3 \cdot 3 - 1)^2} + \frac{1}{3(3-2)^2} + \left[ \frac{1}{3 \cdot 3 - 1} - \frac{1}{3(3-2)} \right]^2 \right\} \sqrt[3]{\frac{3 \cdot 3 - 1}{3-2}}</math></p>	1A 1A 1A 1M 1M	For using (a)
<p><u>Alternative Solution</u></p> <p>When <math>x=3</math>, <math>y=2</math> and so <math>\frac{dy}{dx} = \frac{-5}{12}</math>.</p> <p>By (a), <math>\frac{1}{y} \frac{d^2y}{dx^2} - \frac{1}{y^2} \frac{dy}{dx} \frac{dy}{dx} = \frac{-3}{(3x-1)^2} + \frac{1}{3(x-2)^2}</math></p> <p>When <math>x=3</math>, <math>\frac{1}{2} \frac{d^2y}{dx^2} - \frac{1}{2^2} \cdot \frac{-5}{12} \cdot \frac{-5}{12} = \frac{-3}{(3 \cdot 3 - 1)^2} + \frac{1}{3(3-2)^2}</math></p>	1A 1M 1M	For both $y$ and $\frac{dy}{dx}$ For chain rule
<p>i.e. <math>\frac{d^2y}{dx^2} = \frac{95}{144}</math></p>	1A	OR 0.6597
	(6)	

Solution	Marks	Remarks
<p>5. (a) <math>\frac{dy}{dx} = e^{2x}</math>  <math>y = \frac{1}{2}e^{2x} + C</math>            Since <math>A(0, 1)</math> lies on <math>S</math>, we have <math>1 = \frac{1}{2}e^{2(0)} + C</math>.            i.e. <math>C = \frac{1}{2}</math>            Hence the equation of <math>S</math> is <math>y = \frac{1}{2}e^{2x} + \frac{1}{2}</math>.</p> <p>(b) At <math>A(0, 1)</math>, <math>\frac{dy}{dx} = e^{2(0)} = 1</math>.            Hence the equation of <math>L</math> is <math>y - 1 = 1(x - 0)</math>.            i.e. <math>y = x + 1</math></p> <p>(c) The area of the region bounded by <math>S</math>, <math>L</math> and the line <math>x = 1</math>  <math>= \int_0^1 \left[ \left( \frac{1}{2}e^{2x} + \frac{1}{2} \right) - (x + 1) \right] dx</math>  <math>= \left[ \frac{1}{4}e^{2x} - \frac{1}{2}x^2 - \frac{1}{2}x \right]_0^1</math>  <math>= \frac{e^2 - 5}{4}</math></p>	<p>1A 1M 1A 1M 1A 1M 1A 1A (7)</p>	 <p>1M for <math>A = \int_0^1 (y_1 - y_2) dx</math> OR 0.5973</p>
<p>6. (a) Let <math>X</math> be the weight of a student. The sample mean <math>\bar{X} \sim N\left(67, \frac{15^2}{36}\right)</math>.</p> $P(\bar{X} > 70) = P\left(Z > \frac{70 - 67}{\frac{15}{6}}\right)$ $= P(Z > 1.2)$ $\approx 0.1151$ <p>(b) The sample proportion is <math>\frac{9}{36} = 0.25</math>.            An approximate 95% confidence interval for the proportion  <math>\approx \left(0.25 - 1.96 \times \sqrt{\frac{0.25 \times 0.75}{36}}, 0.25 + 1.96 \times \sqrt{\frac{0.25 \times 0.75}{36}}\right)</math>  <math>\approx (0.1085, 0.3915)</math></p>	<p>1M 1A 1A 1M 1A (5)</p>	
<p>7. (a) <math>\frac{e^{-\lambda}}{0!} = 0.1653</math>  <math>\lambda = -\ln 0.1653</math>  <math>\approx 1.8</math></p> <p>(b) <math>P(\text{no. of goals in a match} &lt; 3) = \frac{e^{-1.8}}{0!} + \frac{e^{-1.8}(1.8)}{1!} + \frac{e^{-1.8}(1.8)^2}{2!}</math>  <math>\approx 0.7306</math></p>	<p>1A 1M 1A</p>	

Solution	Marks	Remarks
(c) The number of goals scored in two matches by the team $\sim \text{Po}(3.6)$ . $\therefore P(\text{no. of goals in two matches} < 3)$ $= \frac{e^{-3.6}}{0!} + \frac{e^{-3.6}(3.6)}{1!} + \frac{e^{-3.6}(3.6)^2}{2!}$	1M	
<u>Alternative Solution</u> $P(\text{no. of goals in two matches} < 3)$ $= P(0, 0) + P(0, 1) + P(1, 0) + P(1, 1) + P(0, 2) + P(2, 0)$ $= \left(\frac{e^{-1.8}}{0!}\right)^2 + 2\left(\frac{e^{-1.8}}{0!}\right)\left[\frac{e^{-1.8}(1.8)}{1!}\right] + \left[\frac{e^{-1.8}(1.8)}{1!}\right]^2 + 2\left(\frac{e^{-1.8}}{0!}\right)\left[\frac{e^{-1.8}(1.8)^2}{2!}\right]$	1M	
$\approx 0.3027$	1A	
	(5)	
8. (a) $P(X=1) + P(X=3) + \dots + P(X=13) = 1$ $0.1 + a + 0.25 + 0.15 + b + 0.05 = 1$ $a + b = 0.45$ ----- (1) $E(X) = 5.5$ $1 \times 0.1 + 3a + 4 \times 0.25 + 6 \times 0.15 + 9b + 13 \times 0.05 = 5.5$ $a + 3b = 0.95$ ----- (2) Solving (1) and (2), we get $a = 0.2$ and $b = 0.25$ .	1M	
	1M	
	1A	For both
(b) (i) $P(F \cap G) = 0.25 + 0.15$ $= 0.4$	1A	
(ii) $P(F) \times P(G) = (0.25 + 0.15 + 0.25 + 0.05)(0.1 + 0.2 + 0.25 + 0.15)$ $= 0.49$ $\neq P(F \cap G)$	1A	
<u>Alternative Solution 1</u> $P(F G) = \frac{P(F \cap G)}{P(G)}$ $= \frac{0.4}{0.1 + 0.2 + 0.25 + 0.15}$ $\approx 0.571428571$ $P(F) = 0.25 + 0.15 + 0.25 + 0.05$ $= 0.7$ $\neq P(F G)$	1A	
<u>Alternative Solution 2</u> $P(G F) = \frac{P(F \cap G)}{P(F)}$ $= \frac{0.4}{0.25 + 0.15 + 0.25 + 0.05}$ $\approx 0.571428571$ $P(G) = 0.1 + 0.2 + 0.25 + 0.15$ $= 0.7$ $\neq P(G F)$	1A	
Hence, $F$ and $G$ are not independent.	1	
	(6)	

Solution	Marks	Remarks
<p>9. (a) Let <math>X</math> be the score of a student who had revised.</p> $P(X \geq 43) = P\left(Z \geq \frac{43-59}{10}\right)$ $= P(Z \geq -1.6)$ $\approx 0.9452$ <p>Let <math>Y</math> be the score of a student who had not revised.</p> $P(Y \geq 43) = P\left(Z \geq \frac{43-35.2}{12}\right)$ $= P(Z \geq 0.65)$ $\approx 0.2578$ $\therefore P(\text{pass the test}) \approx 0.73 \times 0.9452 + 0.27 \times 0.2578$ $= 0.759602$ <p>(b) <math>P(\text{a student had not revised for the test} \mid \text{he passed the test})</math></p> $= \frac{0.27 \times 0.2578}{0.759602}$ $\approx 0.091634829$ $\approx 0.0916$ <p>(c) <math>P(4 \text{ students had not revised for the test among } 10 \text{ passed students})</math></p> $\approx C_6^{10} (0.091634829)^4 (1 - 0.091634829)^6$ $\approx 0.0083$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>(7)</p>	<p>Either one</p> <p>OR 0.7596</p>
<p>10. (a) (i) <math>I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} dt</math></p> $= \frac{1}{2} \cdot \frac{4-1}{6} \left[ \frac{1}{\sqrt{1}} e^{\frac{-1}{2}} + \frac{1}{\sqrt{4}} e^{\frac{-4}{2}} + 2 \left( \frac{1}{\sqrt{1.5}} e^{\frac{-1.5}{2}} + \frac{1}{\sqrt{2}} e^{\frac{-2}{2}} + \frac{1}{\sqrt{2.5}} e^{\frac{-2.5}{2}} \right. \right.$ $\left. \left. + \frac{1}{\sqrt{3}} e^{\frac{-3}{2}} + \frac{1}{\sqrt{3.5}} e^{\frac{-3.5}{2}} \right) \right]$ $\approx 0.692913377$ $\approx 0.6929$ <p>(ii) <math>\frac{d}{dt} \left( \frac{-1}{t^2} e^{\frac{-t}{2}} \right) = \frac{-1}{2} t^{-3} e^{\frac{-t}{2}} + t^{-2} \cdot \frac{-1}{2} e^{\frac{-t}{2}}</math></p> $= \frac{-1}{2} e^{\frac{-t}{2}} \left( \frac{-3}{t^2} + \frac{-1}{t^2} \right)$ $\frac{d^2}{dt^2} \left( \frac{-1}{t^2} e^{\frac{-t}{2}} \right) = \frac{-1}{2} \left[ e^{\frac{-t}{2}} \left( \frac{-3}{2} t^{-5} + \frac{-1}{2} t^{-3} \right) + \frac{-1}{2} e^{\frac{-t}{2}} \left( \frac{-3}{t^2} + \frac{-1}{t^2} \right) \right]$ $= \frac{1}{4} e^{\frac{-t}{2}} \left( 3t^{-5} + 2t^{-3} + t^{-1} \right)$ $> 0 \quad \text{for } 1 \leq t \leq 4.$ <p>Hence the estimation in (i) is an over-estimate.</p>	<p>1M</p> <p>1A</p> <p>1M+1A</p> <p>1M+1A</p> <p>1</p> <p>(7)</p>	

Solution	Marks	Remarks
(b) Let $t = x^2$ . $dt = 2xdx$ When $t = 1$ , $x = 1$ ; when $t = 4$ , $x = 2$ . $I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} dt$ $= \int_1^2 \frac{1}{x} e^{\frac{-x^2}{2}} 2xdx$ $= 2 \int_1^2 e^{\frac{-x^2}{2}} dx$	1M } 1A  1 (3)	
(c) $2 \int_1^2 e^{\frac{-x^2}{2}} dx < 0.692913377$ $2\sqrt{2\pi} \int_1^2 \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx < 0.692913377$ $2\sqrt{2\pi}(0.4772 - 0.3413) < 0.692913377$ $\pi < 3.249593152$ $\therefore \pi < 3.25$	1M  1A  1 (3)	For 0.4772 and 0.3413
11. (a) When $t = 35$ , the intensity increased to a maximum and therefore $\frac{dR}{dt} = 0$ . $\frac{a(30-35)+10}{(35-35)^2+b} = 0$ $a = 2$	1A 1A (2)	
(b) $\frac{dR}{dt} = \frac{2(30-t)+10}{(t-35)^2+b}$ Let $u = (t-35)^2 + b$ . $du = 2(t-35)dt$ $R = \int \frac{-2t+70}{(t-35)^2+b} dt$ $= \int \frac{-2t+70}{u} \frac{du}{2(t-35)}$ $= -\ln u  + C$ $= -\ln[(t-35)^2 + b] + C$ $R _{t=T} = R _{t=0}$ $-\ln[(T-35)^2 + b] + C = -\ln[(0-35)^2 + b] + C$ $(T-35)^2 = 35^2$ $T = 70 \text{ or } 0 \text{ (rejected)}$	1M  1A 1M 1A (4)	

Solution	Marks	Remarks								
<p>(c) <math>R _{t=40} - R _{t=41} = \ln \frac{61}{50}</math></p> $-\ln[(40-35)^2 + b] + C - \{-\ln[(41-35)^2 + b] + C\} = \ln \frac{61}{50}$ $-\ln(25 + b) + \ln(36 + b) = \ln \frac{61}{50}$ $\ln \frac{36 + b}{25 + b} = \ln \frac{61}{50}$ $b = 25$ $\therefore R = -\ln[(t-35)^2 + 25] + C$ $R _{t=35} = 6$ $-\ln[(35-35)^2 + 25] + C = 6$ $C = 6 + \ln 25$ <p>i.e. <math>R = -\ln[(t-35)^2 + 25] + 6 + \ln 25</math></p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>									
(4)										
<p>(d) <math>\frac{dR}{dt} = \frac{2(30-t)+10}{(t-35)^2 + 25}</math></p> $= \frac{70-2t}{t^2 - 70t + 1250}$ $\frac{d^2R}{dt^2} = \frac{(t^2 - 70t + 1250)(-2) - (70-2t)(2t-70)}{(t^2 - 70t + 1250)^2}$ $= \frac{2t^2 - 140t + 2400}{(t^2 - 70t + 1250)^2}$ <p>When the rate of change of the radiation intensity attains its greatest value, <math>\frac{d^2R}{dt^2} = 0</math>.</p> $2t^2 - 140t + 2400 = 0$ $t = 30 \text{ or } 40 \text{ (rejected)}$	<p>1M+1A</p>									
<table border="1" data-bbox="134 1355 613 1469"> <thead> <tr> <th><math>t</math></th> <th><math>0 \leq t &lt; 30</math></th> <th><math>t = 30</math></th> <th><math>30 &lt; t \leq 35</math></th> </tr> </thead> <tbody> <tr> <td><math>\frac{d^2R}{dt^2}</math></td> <td>+ve</td> <td>0</td> <td>-ve</td> </tr> </tbody> </table>	$t$	$0 \leq t < 30$	$t = 30$	$30 < t \leq 35$	$\frac{d^2R}{dt^2}$	+ve	0	-ve	<p>1M</p>	
$t$	$0 \leq t < 30$	$t = 30$	$30 < t \leq 35$							
$\frac{d^2R}{dt^2}$	+ve	0	-ve							
<p>Hence, the rate of change of the radiation intensity would attain its greatest value when <math>t = 30</math>.</p>	<p>1A</p>									
(4)										



Solution	Marks	Remarks
12. (a) (i) The sample mean = $\frac{56 + \dots + 50}{16}$ = 51.5625	1A	
A 90% confidence interval	1M+1A	
$\approx \left( 51.5625 - 1.645 \times \frac{9}{\sqrt{16}}, 51.5625 + 1.645 \times \frac{9}{\sqrt{16}} \right)$	1A	OR (47.8613, 55.2638)
= (47.86125, 55.26375)		
(ii) Let $n$ be the sample size.		
$\therefore 2 \left( 1.645 \cdot \frac{9}{\sqrt{n}} \right) < 6$	1M	
$n > 24.354225$	1A	
Hence, the least sample size is 25.	1A	
	(7)	
(b) (i) P(a tourist waits for more than 65 minutes)		
$= P \left( Z > \frac{65 - 51.5}{9} \right)$	1M	
$= P(Z > 1.5)$		
$\approx 0.0668$	1A	
P(less than 2 coupons are sent to the first 10 tourists interviewed)		
$\approx (1 - 0.0668)^{10} + C_1^{10} (1 - 0.0668)^9 (0.0668)$	1M	
$\approx 0.8594$	1A	
(ii) P(the 5th coupon is sent to the 20th tourist interviewed)		
$\approx C_4^{19} (1 - 0.0668)^{15} (0.0668)^4 \cdot 0.0668$	1M	
$\approx 0.0018$	1A	
	(6)	

Solution	Marks	Remarks
13. (a) P(at least 2 drunk drivers are prosecuted) $= 1 - e^{-2.3} - e^{-2.3}(2.3)$ $\approx 0.669145815$ $\approx 0.6691$	1A  1A	
	(2)	
(b) P( $\leq 4$ drunk drivers are prosecuted   at least 2 drunk drivers are prosecuted) $\frac{e^{-2.3} \left( \frac{2.3^2}{2!} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!} \right)}{0.669145815}$ $\approx 0.8748$	1M+1M 1A	1M for Poisson 1M for conditional prob
	(3)	
(c) (i) P(the third night was the 1st night to have $\geq 2$ drunk drivers prosecuted) $\approx (1 - 0.669145815)^2 (0.669145815)$ $\approx 0.0732$	1M 1A	
(ii) P( $\geq 2$ drunk drivers prosecuted in each night and totally 10 prosecuted) $= C_2^3 \left( e^{-2.3} \frac{2.3^2}{2!} \right)^2 \left( e^{-2.3} \frac{2.3^6}{6!} \right) + 3! \left( e^{-2.3} \frac{2.3^2}{2!} \right) \left( e^{-2.3} \frac{2.3^3}{3!} \right) \left( e^{-2.3} \frac{2.3^5}{5!} \right)$ $+ C_2^3 \left( e^{-2.3} \frac{2.3^2}{2!} \right) \left( e^{-2.3} \frac{2.3^4}{4!} \right)^2 + C_2^3 \left( e^{-2.3} \frac{2.3^3}{3!} \right)^2 \left( e^{-2.3} \frac{2.3^4}{4!} \right)$ $\approx 0.0471$	} 1M+1M  1A	1M for any one case 1M for all cases
	(5)	