Paper 1

Solution	Marks	Remarks
1. $\dfrac{m^9}{(m^3n^{-7})^5}$		
$=\dfrac{m^9}{m^{15}n^{-35}}$	1M	for $(a^h)^k=a^{hk}$ or $(ab)^\ell=a^\ell b^\ell$
$=\dfrac{n^{35}}{m^{15-9}}$	1M	for $\dfrac{c^p}{c^q}=c^{p-q}$ or $d^{-r}=\dfrac{1}{d^r}$
$=\dfrac{n^{35}}{m^6}$	1A	
	----------(3)	
2. $\dfrac{4a+5b-7}{b}=8$		
$4a+5b-7=8b$	1M	
$4a-7=8b-5b$	1M	for putting b on one side
$4a-7=3b$		
$b=\dfrac{4a-7}{3}$	1A	or equivalent
$\dfrac{4a+5b-7}{b}=8$		
$\dfrac{4a-7}{b}+5=8$	1M	
$\dfrac{4a-7}{b}=8-5$	1M	for putting constants on one side
$\dfrac{4a-7}{b}=3$		
$b=\dfrac{4a-7}{3}$	1A	or equivalent
	----------(3)	
3. The required probability		
$=\dfrac{1+2+3}{(4)(5)}$	1M+1M	1M for numerator 1M for denominator
$=\dfrac{6}{20}$		
$=\dfrac{3}{10}$	1A	0.3
	----------(3)	

Solution	Marks	Remarks
4. (a) $x^3+x^2y-7x^2$		
$=x^2(x+y-7)$	1A	or equivalent
(b) $x^3+x^2y-7x^2-x-y+7$		
$=x^2(x+y-7)-x-y+7$	1M	for using the result of (a)
$=x^2(x+y-7)-(x+y-7)$		
$=(x^2-1)(x+y-7)$	1M	
$=(x-1)(x+1)(x+y-7)$	1A	or equivalent
	----------(4)	
5. (a) $\frac{7-3x}{5}\le 2(x+2)$		
$7-3x\le 10(x+2)$		
$7-3x\le 10x+20$		
$-13\le 13x$		
$x\ge -1$	1A	
$4x-13>0$		
$x>\frac{13}{4}$	1A	$x>3.25$
Thus, the required range is $x>\frac{13}{4}$.	1M	
(b) 4	1A	
	----------(4)	

Solution	Marks	Remarks
6. (a) The selling price of the book		
$= 250(1+20\%)$	1M	
$= \$300$	1A	
(b) Let $\$x$ be the marked price of the book.		
$(1-25\%)x = 300$	1M	
$x = \dfrac{300}{75\%}$		
$x = 400$	1A	
Thus, the marked price of the book is $\$400$.		
	----------(4)	
7. Let x be the number of apples owned by Billy.		
Then, the number of apples owned by Ada is $4x$.	1A	
$4x-12 = x+12$	1A+1M	
$3x = 24$		
$x = 8$		
Thus, the total number of apples owned by Ada and Billy is 40 .	1A	
Let x and y be the numbers of apples owned by Ada and Billy respectively.		
So, we have $x = 4y$ and $x-12 = y+12$.	1A+1A	
Therefore, we have $4y-12 = y+12$.	1M	for getting a linear equation in x or y only
Hence, we have $3y = 24$.		
Solving, we have $x = 32$ and $y = 8$.		
Thus, the total number of apples owned by Ada and Billy is 40 .	1A	
Let x be the total number of apples owned by Ada and Billy.		
Then, the numbers of apples owned by Ada and Billy are $\left(\dfrac{x}{2}+12\right)$ and $\left(\dfrac{x}{2}-12\right)$ respectively.	1A	for both correct
$\dfrac{x}{2}+12 = 4\left(\dfrac{x}{2}-12\right)$	1A+1M	
$\dfrac{x}{2}+12 = 2x-48$		
$3x = 120$		
$x = 40$	1A	
Thus, the total number of apples owned by Ada and Billy is 40 .		
The total number of apples owned by Ada and Billy		
$= \dfrac{(12-(-12))(4+1)}{4-1}$	1M+1A+1A	1M for fraction + 1A for numerator + 1A for denominator
$= \dfrac{(24)(5)}{3}$		
$= 40$	1A	
	----------(4)	

Solution	Marks	Remarks
8. Note that $\angle ABD = \angle ADB = 58°$.	1M	either one (covers the lines from $\angle ABD = \angle ADB = 58°$ to $\angle BCE = \angle ADB = 58°$)
Also note that $\angle ABC + \angle ADC = 180°$.		
So, we have $58° + 25° + 58° + \angle BDC = 180°$.		
Therefore, we have $\angle BDC = 39°$.	1A	
Further note that $\angle BCE = \angle ADB = 58°$.		
$\angle BEC$		
$= \dfrac{180° - \angle BCE}{2}$	1M	
$= \dfrac{180° - 58°}{2}$		
$= 61°$		
$\angle ABE$		
$= \angle BEC - \angle BAC$	1M	
$= \angle BEC - \angle BDC$		
$= 61° - 39°$		
$= 22°$	1A	
Note that $\angle ABD = \angle ADB = 58°$ and $\angle ACB = \angle ADB = 58°$.	1M	for either one
$\angle CBE$		
$= \angle BEC$		
$= \dfrac{180° - \angle BCE}{2}$	1M	
$= \dfrac{180° - \angle ACB}{2}$		
$= \dfrac{180° - 58°}{2}$		
$= 61°$		
$\angle ABE$		
$= \angle ABD + \angle CBD - \angle CBE$		
$= 58° + 25° - 61°$		
$= 22°$	1A	
$\angle BDC$		
$= \angle BAE$		
$= \angle BEC - \angle ABE$	1M	
$= 61° - 22°$		
$= 39°$	1A	
	----------(5)	
9. (a) Let $\theta°$ be the angle of the sector.		
$\dfrac{\theta}{360}\left(\pi(12^2)\right) = 30\pi$	1M	
$\theta = 75$	1A	
Thus, the angle of the sector is $75°$.		
(b) The required perimeter		
$= \dfrac{75}{360}\left(2\pi(12)\right) + 2(12)$	1M+1M	
$= (5\pi + 24)$ cm	1A	
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $S = a + bn$, where a and b are non-zero constants.	1A	
So, we have $a + b(10) = 10\ 600$ and $a + b(6) = 9\ 000$.	1M	for either substitution
Solving, we have $a = 6\ 600$ and $b = 400$.	1A	for both correct
The required income		
$= 6\ 600 + 400(20)$		
$= \$14\ 600$	1A	
	----------(4)	
(b) $6\ 600 + 400n = 18\ 000$	1M	
$400n = 11\ 400$		
$n = 28.5$		
Note that 28.5 is not an integer.		
Thus, it is not possible that Susan's income in that month is $\$18\ 000$.	1A	f.t.
	----------(2)	
11. (a) $k = -5$	1A	
$\mathrm{f}(3) = 0$	1M	
$(3-2)^2(3+h) - 5 = 0$		
$h = 2$	1A	
	----------(3)	
(b) $\mathrm{f}(x) = 0$		
$(x-2)(x-2)(x+2) - 5 = 0$		
$x^3 - 2x^2 - 4x + 3 = 0$	1A	
$(x-3)(x^2 + x - 1) = 0$	1M	for $(x-3)(ax^2 + bx + c)$
$x = 3$ or $x = \dfrac{-1 \pm \sqrt{5}}{2}$		
Note that both $\dfrac{-1+\sqrt{5}}{2}$ and $\dfrac{-1-\sqrt{5}}{2}$ are not integers.		
Thus, the claim is disagreed.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
12. (a) The mean		
$= 55$ kg	1A	
The median		
$= 52$ kg	1A	
The range		
$= 79 - 40$		
$= 39$ kg	1A	
	----------(3)	
(b) Let a kg and b kg be the weights of these two students, where $a \leq b$.		
Note that $\frac{a+b+55(20)}{22} = 55 + 1$.	1M	
Therefore, we have $a + b = 132$.		
Since the range is increased by 1 kg , the new range is 40 kg .		
There are two cases.		
Case 1: $a = 39$	1M	either one
Since $a + b = 132$, we have $b = 93$.		
Therefore, the new range is 54 kg .		
It is impossible.		
Case 2: $40 \leq a \leq 80$		
Under this case, we have $b = 80$.	1A	
Since $a + b = 132$, we have $a = 52$.	1A	
Thus, the weights of these two students are 52 kg and 80 kg .		
	----------(4)	

Solution	Marks	Remarks
13. (a) $AB = BC$ (property of square)		
$AE = BF$ (given)		
$\angle ABE = 90° = \angle BCF$ (property of square)		
$\Delta ABE \cong \Delta BCF$ (RHS)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	----------(2)	
(b) By (a), we have $\angle BAE = \angle CBF$.	1M	
$\angle AEB$		
$= 180° - \angle ABE - \angle BAE$	1M	either one
$= 180° - 90° - \angle BAE$		
$= 90° - \angle BAE$		
$\angle BGE$		
$= 180° - \angle CBF - \angle AEB$		
$= 180° - \angle BAE - (90° - \angle BAE)$		
$= 90°$		
Thus, ΔBGE is a right-angled triangle.	1A	f.t.
By (a), we have $\angle BAE = \angle CBF$.	1M	
Note that $\angle AEB = \angle DAE$.		
Since $\angle BAE + \angle DAE = 90°$, we have $\angle CBF + \angle AEB = 90°$.		
Also note that $\angle CBF + \angle AEB + \angle BGE = 180°$.	1M	
So, we have $\angle BGE = 90°$.		
Thus, ΔBGE is a right-angled triangle.	1A	f.t.
	----------(3)	
(c) By (a), we have $BE = CF = 15$ cm .		
BG		
$= \sqrt{BE^2 - EG^2}$	1M	
$= \sqrt{15^2 - 9^2}$		
$= 12$ cm	1A	
	----------(2)	

Solution	Marks	Remarks
14. (a) (i) The mid-point of PQ		
$=(-5,11)$		
The slope of PQ		
$=\dfrac{23-(-1)}{-14-4}$	1M	
$=\dfrac{-4}{3}$		
The equation of L is		
$y-11=\dfrac{3}{4}(x-(-5))$	1M	
$3x-4y+59=0$	1A	or equivalent
The equation of L is		
$(x-4)^2+(y+1)^2=(x+14)^2+(y-23)^2$	1M+1M	
$x^2-8x+16+y^2+2y+1=x^2+28x+196+y^2-46y+529$		
$36x-48y+708=0$		
$3x-4y+59=0$	1A	or equivalent
(ii) Let k be the y-coordinate of G.		
By (a)(i), we have $3h-4k+59=0$.		
So, we have $k=\dfrac{3h+59}{4}$.	1M	
The equation of C is		
$(x-h)^2+(y-k)^2=(4-h)^2+(-1-k)^2$	1M	
$x^2+y^2-2hx-2\left(\dfrac{3h+59}{4}\right)y+8h-2\left(\dfrac{3h+59}{4}\right)-17=0$		
$2x^2+2y^2-4hx-(3h+59)y+13h-93=0$	1	
Denote the circle $2x^2+2y^2-4hx-(3h+59)y+13h-93=0$ by C'.		
The coordinates of the centre of C' are $\left(h,\dfrac{3h+59}{4}\right)$.	1M	
Let k be the y-coordinate of G.		
By (a)(i), we have $3h-4k+59=0$.		
So, we have $k=\dfrac{3h+59}{4}$.	1M	
Therefore, the centre of C' is G.		
Also note that $2(4)^2+2(-1)^2-4h(4)-(3h+59)(-1)+13h-93=0$		
and $2(-14)^2+2(23)^2-4h(-14)-(3h+59)(23)+13h-93=0$.		
Hence, C' is the circle which is centred at G and passes through P and Q.	1	
Thus, the equation of C is $2x^2+2y^2-4hx-(3h+59)y+13h-93=0$.		
	----------(6)	

Solution	Marks	Remarks
(b) Denote the circle which passes through P, Q and R by C. Note that the centre of C lies on the perpendicular bisector of PQ. Let h be the x-coordinate of the centre of C. By (a)(ii), we have $2(26)^2+2(43)^2-4h(26)-(3h+59)(43)+13h-93=0$.	1M	for using (a)(ii)
So, we have $h=11$. Hence, the equation of C is $x^2+y^2-22x-46y+25=0$. The required diameter $=2\left(\sqrt{\left(\frac{22}{2}\right)^2+\left(\frac{46}{2}\right)^2-25}\right)$	1M	
$=2\sqrt{625}$ $=50$	1A	
Denote the circle which passes through P, Q and R by C. Note that the centre of C lies on the perpendicular bisector of PQ. Let (a,b) be the coordinates of the centre of C. Then, we have $\begin{cases}3a-4b+59=0\\(a-4)^2+(b+1)^2=(a-26)^2+(b-43)^2\end{cases}$.	1M	
Hence, we have $\begin{cases}3a-4b+59=0\\a+2b-57=0\end{cases}$. Solving, we have $a=11$ and $b=23$. The required diameter $=2\sqrt{(11-4)^2+(23+1)^2}$	1M	
$=2\sqrt{625}$ $=50$	1A	
	----------(3)	

Solution	Marks	Remarks
15. (a) Let x marks be the score of David in the Mathematics examination.		
$\frac{x-66}{12}=-0.5$	1M	
$x=66-(0.5)(12)$		
$x=60$	1A	
Thus, the score of David in the Mathematics examination is 60 marks.	----------(2)	
(b) The standard score of David in the Science examination		
$=\frac{49-52}{10}$		
$=-0.3$	1A	
>-0.5		
Relative to other students, David performs better in the Science examination than in the Mathematics examination.		
Thus, the claim is correct.	1A ----------(2)	f.t.
16. (a) The required probability		
$=\frac{C_2^5 C_2^9}{C_4^{14}}$	1M	for numerator
$=\frac{360}{1\,001}$	1A	r.t. 0.360
The required probability		
$=6\left(\frac{5}{14}\right)\left(\frac{4}{13}\right)\left(\frac{9}{12}\right)\left(\frac{8}{11}\right)$	1M	for $6p_1\ p_2\ p_3\ p_4$
$=\frac{360}{1\,001}$	1A	r.t. 0.360
	----------(2)	
(b) The required probability		
$=\frac{360}{1\,001}+\frac{C_3^5 C_1^9}{C_4^{14}}+\frac{C_4^5}{C_4^{14}}$	1M	for $(a)+p_5+p_6$
$=\frac{5}{11}$	1A	r.t. 0.455
The required probability		
$=\frac{360}{1001}+4\left(\frac{5}{14}\right)\left(\frac{4}{13}\right)\left(\frac{3}{12}\right)\left(\frac{9}{11}\right)+\left(\frac{5}{14}\right)\left(\frac{4}{13}\right)\left(\frac{3}{12}\right)\left(\frac{2}{11}\right)$	1M	for $(a)+p_7+p_8$
$=\frac{5}{11}$	1A	r.t. 0.455
The required probability		
$=1-\frac{C_4^9}{C_4^{14}}-\frac{C_1^5 C_3^9}{C_4^{14}}$	1M	for $1-p_9-p_{10}$
$=\frac{5}{11}$	1A	r.t. 0.455
	----------(2)	

Solution	Marks	Remarks
17. (a) $A(1)+A(2)+A(3)+\cdots+A(n)$		
$=-1+3+7+\cdots+(4n-5)$		
$=\frac{n}{2}((-1)+(4n-5))$	1M	
$=n(2n-3)$	1A	or equivalent
	----------(2)	
(b) $\log(B(1)B(2)B(3)\cdots B(n)) \le 8\,000$		
$\log B(1)+\log B(2)+\log B(3)+\cdots+\log B(n) \le 8\,000$	1M	
Note that $\log B(k)=A(k)$ for all positive integers k.		
$A(1)+A(2)+A(3)+\cdots+A(n) \le 8\,000$		
$n(2n-3) \le 8\,000$	1M	
$2n^2-3n-8\,000 \le 0$		
$(n-64)(2n+125) \le 0$		
$\frac{-125}{2} \le n \le 64$		
Thus, the greatest value of n is 64.	1A	
$\log(B(1)B(2)B(3)\cdots B(n)) \le 8\,000$		
$\log(10^{-1}10^{3}10^{7}\cdots 10^{4n-5}) \le 8\,000$		
$\log(10^{-1+3+7+\cdots+(4n-5)}) \le 8\,000$	1M	
$\log(10^{n(2n-3)}) \le 8\,000$		
$n(2n-3) \le 8\,000$	1M	
$2n^2-3n-8\,000 \le 0$		
$(n-64)(2n+125) \le 0$		
$\frac{-125}{2} \le n \le 64$		
Thus, the greatest value of n is 64.	1A	
	----------(3)	

Solution	Marks	Remarks
18. (a) $(-4k)^2-4(2)(3k^2+5)$	1M	
$=16k^2-24k^2-40$		
$=-8k^2-40$		
<0		
Thus, the graph of $y=\mathrm{f}(x)$ does not cut the x-axis.	1A	f.t.
	----------(2)	
(b) $\mathrm{f}(x)$		
$=2x^2-4kx+3k^2+5$		
$=2(x^2-2kx)+3k^2+5$		
$=2(x^2-2kx+k^2-k^2)+3k^2+5$	1M	
$=2(x-k)^2+k^2+5$	1A	
Thus, the coordinates of the vertex are (k,k^2+5) .	1M	
	----------(3)	
(c) By (b), the coordinates of the vertex of the graph of $y=2-\mathrm{f}(x)$ are $(k,-k^2-3)$.	1M	
When S and T are nearest to each other, the coordinates of S and T are (k,k^2+5) and $(k,-k^2-3)$ respectively.		
In this case, ST is a vertical line.	1M	
So, the perpendicular bisector of ST is a horizontal line.		
The y-coordinate of the circumcentre of ΔOST		
$=\dfrac{(k^2+5)+(-k^2-3)}{2}$	1M	
$=1$		
$\neq 0$		
Therefore, the circumcentre of ΔOST does not lie on the x-axis.		
Thus, the claim is incorrect.	1A	f.t.
Assume that when S and T are nearest to each other, the circumcentre of ΔOST lies on the x-axis.		
In this case, the coordinates of S and T are (k,k^2+5) and $(k,-k^2-3)$ respectively.	1M	
Let $(r,0)$ be the coordinates of the circumcentre R of ΔOST .		
RS	1M	either one
$=\sqrt{(r-k)^2+(0-(k^2+5))^2}$		
$=\sqrt{(r-k)^2+(k^2+5)^2}$		
RT		
$=\sqrt{(r-k)^2+(0-(-k^2-3))^2}$		
$=\sqrt{(r-k)^2+(k^2+3)^2}$		
So, we have $RS\neq RT$.	1M	
It is impossible.		
Thus, the claim is incorrect.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
19. (a) (i) By cosine formula,		
$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$	1M	
$AC^2 = 40^2 + 24^2 - 2(40)(24)\cos 80^\circ$		
$AC \approx 42.92546446$ cm		
$AC \approx 42.9$ cm	1A	r.t. 42.9 cm
Thus, the distance between A and C is 42.9 cm .		
(ii) By sine formula,		
$\frac{\sin\angle ACB}{AB} = \frac{\sin\angle ABC}{AC}$	1M	
$\frac{\sin\angle ACB}{40} \approx \frac{\sin 80^\circ}{42.92546446}$		
$\angle ACB \approx 66.59081487^\circ$ or $\angle ACB \approx 113.4091851^\circ$ (rejected)		
$\angle ACB \approx 66.6^\circ$	1A	r.t. 66.6°
(iii) $\angle CAD = 180^\circ - 2(\angle BCD - \angle ACB)$		
$23.18162974^\circ < \angle CAD < 103.1816297^\circ$		
The area of the paper card		
$= 2\left(\frac{1}{2}(40)(24)\sin 80^\circ\right) + \frac{1}{2}AC^2\sin\angle CAD$	1M	
$= 960\sin 80^\circ + \frac{1}{2}AC^2\sin\angle CAD$		
Note that $960\sin 80^\circ$ is a constant and $\frac{1}{2}AC^2\sin\angle CAD$ varies as $\sin\angle CAD$.	1M	
Also note that the area of the paper card is the greatest when $\angle CAD = 90^\circ$.		
Define $\alpha = 45^\circ + \angle ACB$.		
Then, we have $\alpha \approx 111.59081487^\circ$.		
When $\angle BCD$ increases from 105° to α , the area of the the paper card increases. When $\angle BCD$ increases from α to 145° , the area of the the paper card decreases.	1A	f.t.
	----------(7)	
(b) $\angle ACD = \angle BCD - \angle ACB$		
$\angle ACD \approx 65.40918513^\circ$		
$\cos\angle ACD = \frac{\frac{CD}{2}}{AC}$	1M	
$CD \approx 35.72557859$ cm		
Let M be the mid-point of CD.		
$AM^2 = AC^2 - CM^2$		
$AM^2 \approx 1523.516258$		
$BM^2 = BC^2 - CM^2$		
$BM^2 \approx 256.9207587$		
By cosine formula,		
$\cos\angle AMB = \frac{AM^2 + BM^2 - AB^2}{2(AM)(BM)}$	1M	
$\angle AMB \approx 81.70890517^\circ$		

Solution	Marks	Remarks
The height of the pyramid $ABCD$ $= BM \sin \angle AMB$ ≈ 15.86121883 cm	1M	accept $BA \sin \angle BAM$
The area of ΔACD $= \frac{1}{2}(CD)(AM)$ ≈ 697.2247927 cm^2	1M	
The volume of the pyramid $ABCD$ $= \frac{1}{3}$(the area of ΔACD)(the height of the pyramid $ABCD$) $\approx 3\,686.278338$ cm^3	1M	
$\approx 3\,690$ cm^3	1A	r.t. $3\,690$ cm^3
$\angle ACD = \angle BCD - \angle ACB$ $\angle ACD \approx 65.40918513°$ $\cos \angle ACD = \frac{\frac{CD}{2}}{AC}$ $CD \approx 35.72557859$ cm	1M	
Let M be the mid-point of CD . $AM^2 = AC^2 - CM^2$ $AM^2 \approx 1523.516258$ $BM^2 = BC^2 - CM^2$ $BM^2 \approx 256.9207587$		
By cosine formula, $\cos \angle ABM = \frac{AB^2 + BM^2 - AM^2}{2(AB)(BM)}$ $\angle ABM \approx 74.92963499°$	1M	
The height of the pyramid $ABCD$ $= AB \sin \angle ABM$ ≈ 38.62428968 cm	1M	accept $AM \sin \angle AMB$
The area of ΔBCD $= \frac{1}{2}(CD)(BM)$ ≈ 286.318146 cm^2	1M	
The volume of the pyramid $ABCD$ $= \frac{1}{3}$(the area of ΔBCD)(the height of the pyramid $ABCD$) $\approx 3\,686.278338$ cm^3	1M	
$\approx 3\,690$ cm^3	1A	r.t. $3\,690$ cm^3
	----------(6)	

Paper 2

Question No.	Key	Question No.	Key
1.	D	26.	C
2.	D	27.	A
3.	A	28.	B
4.	D	29.	A
5.	B	30.	B
6.	A	31.	C
7.	A	32.	A
8.	D	33.	A
9.	B	34.	B
10.	D	35.	C
11.	C	36.	D
12.	D	37.	D
13.	B	38.	B
14.	C	39.	D
15.	C	40.	C
16.	B	41.	B
17.	D	42.	A
18.	A	43.	C
19.	C	44.	D
20.	C	45.	B
21.	B		
22.	A		
23.	C		
24.	A		
25.	B		