2017-DSE
MATH CP
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2017

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

8.30 am – 10.45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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Candidate Number									

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Make y the subject of the formula $k = \dfrac{3x - y}{y}$. (3 marks)

2. Simplify $\dfrac{(m^4 n^{-1})^3}{(m^{-2})^5}$ and express your answer with positive indices. (3 marks)

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3. Factorize

(a) $x^2 - 4xy + 3y^2$,

(b) $x^2 - 4xy + 3y^2 + 11x - 33y$.

(3 marks)

4. There are only two kinds of admission tickets for a theatre: regular tickets and concessionary tickets. The prices of a regular ticket and a concessionary ticket are $\$126$ and $\$78$ respectively. On a certain day, the number of regular tickets sold is 5 times the number of concessionary tickets sold and the sum of money for the admission tickets sold is $\$50\,976$. Find the total number of admission tickets sold that day. (4 marks)

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5. (a) Find the range of values of x which satisfy both $7(x-2) \leq \frac{11x+8}{3}$ and $6-x<5$.

(b) How many integers satisfy both inequalities in (a)?

(4 marks)

6. The coordinates of the points A and B are $(-3,4)$ and $(9,-9)$ respectively. A is rotated anticlockwise about the origin through $90°$ to A' . B' is the reflection image of B with respect to the x-axis.

(a) Write down the coordinates of A' and B' .

(b) Prove that AB is perpendicular to $A'B'$.

(4 marks)

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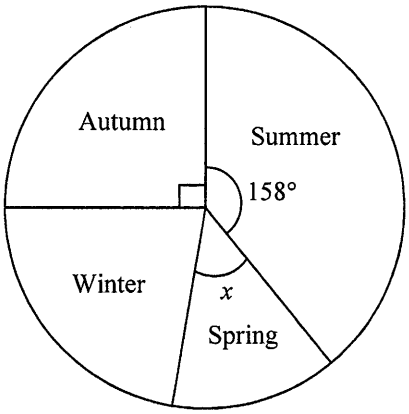
7. The pie chart below shows the distribution of the seasons of birth of the students in a school.



Distribution of the seasons of birth of the students in the school

If a student is randomly selected from the school, then the probability that the selected student was born in spring is $\frac{1}{9}$.

(a) Find x.

(b) In the school, there are 180 students born in winter. Find the number of students in the school.

(4 marks)

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8. It is given that y varies inversely as $\sqrt{x}$. When $x=144$, $y=81$.

(a) Express y in terms of x .

(b) If the value of x is increased from 144 to 324 , find the change in the value of y .

(5 marks)

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9. A bottle is termed *standard* if its capacity is measured as 200 mL correct to the nearest 10 mL .

(a) Find the least possible capacity of a *standard* bottle.

(b) Someone claims that the total capacity of 120 *standard* bottles can be measured as 23.3 L correct to the nearest 0.1 L . Do you agree? Explain your answer.

(5 marks)

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SECTION A(2) (35 marks)

10. In Figure 1, $OPQR$ is a quadrilateral such that $OP = OQ = OR$. OQ and PR intersect at the point S . S is the mid-point of PR .

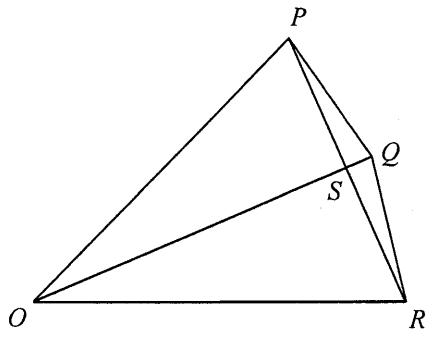

Figure 1

(a) Prove that $\Delta OPS \cong \Delta ORS$. (2 marks)

(b) It is given that O is the centre of the circle which passes through P , Q and R . If $OQ = 6$ cm and $\angle PRQ = 10°$, find the area of the sector $OPQR$ in terms of π . (4 marks)

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11. The stem-and-leaf diagram below shows the distribution of the hourly wages (in dollars) of the workers in a group.

Stem (tens)	Leaf (units)
6	1 1 1 3 4 6 8 9 9
7	a 7 7 8
8	1 b

It is given that the mean and the range of the above distribution are \$70 and \$22 respectively.

(a) Find the median and the standard deviation of the above distribution. (5 marks)

(b) If a worker is randomly selected from the group, find the probability that the hourly wage of the selected worker exceeds \$70 . (2 marks)

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12. A solid metal right prism of base area $84\ cm^2$ and height $20\ cm$ is melted and recast into two similar solid right pyramids. The bases of the two pyramids are squares. The ratio of the base area of the smaller pyramid to the base area of the larger pyramid is $4:9$.

(a) Find the volume of the larger pyramid. (3 marks)

(b) If the height of the larger pyramid is $12\ cm$, find the total surface area of the smaller pyramid. (4 marks)

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13. The coordinates of the points E, F and G are $(-6,5)$, $(-3,11)$ and $(2,-1)$ respectively. The circle C passes through E and the centre of C is G.

(a) Find the equation of C. (2 marks)

(b) Prove that F lies outside C. (2 marks)

(c) Let H be a moving point on C. When H is farthest from F,

(i) describe the geometric relationship between F, G and H;

(ii) find the equation of the straight line which passes through F and H. (3 marks)

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14. Let $f(x) = 6x^3 - 13x^2 - 46x + 34$. When $f(x)$ is divided by $2x^2 + ax + 4$, the quotient and the remainder are $3x + 7$ and $bx + c$ respectively, where a , b and c are constants.

(a) Find a . (3 marks)

(b) Let $g(x)$ be a quadratic polynomial such that when $g(x)$ is divided by $2x^2 + ax + 4$, the remainder is $bx + c$.

(i) Prove that $f(x) - g(x)$ is divisible by $2x^2 + ax + 4$.

(ii) Someone claims that all the roots of the equation $f(x) - g(x) = 0$ are integers. Do you agree? Explain your answer.

(5 marks)

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SECTION B (35 marks)

15. Let a and b be constants. Denote the graph of $y = a + \log_b x$ by G. The x-intercept of G is 9 and G passes through the point $(243, 3)$. Express x in terms of y. (4 marks)

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16. A city adopts a plan to import water from another city. It is given that the volume of water imported in the 1st year since the start of the plan is $1.5\times10^7\ m^3$ and in subsequent years, the volume of water imported each year is 10% less than the volume of water imported in the previous year.

(a) Find the total volume of water imported in the first 20 years since the start of the plan. (2 marks)

(b) Someone claims that the total volume of water imported since the start of the plan will not exceed $1.6\times10^8\ m^3$. Do you agree? Explain your answer. (2 marks)

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17. In a bag, there are 4 green pens, 7 blue pens and 8 black pens. If 5 pens are randomly drawn from the bag at the same time,

(a) find the probability that exactly 4 green pens are drawn; (2 marks)

(b) find the probability that exactly 3 green pens are drawn; (2 marks)

(c) find the probability that not more than 2 green pens are drawn. (2 marks)

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18. The equation of the parabola Γ is $y=2x^2-2kx+2x-3k+8$, where k is a real constant. Denote the straight line $y=19$ by L.

(a) Prove that L and Γ intersect at two distinct points. (3 marks)

(b) The points of intersection of L and Γ are A and B.

(i) Let a and b be the x-coordinates of A and B respectively. Prove that $(a-b)^2=k^2+4k+23$.

(ii) Is it possible that the distance between A and B is less than 4? Explain your answer. (5 marks)

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19. ABC is a thin triangular metal sheet, where $BC = 24$ cm , $\angle BAC = 30°$ and $\angle ACB = 42°$.

(a) Find the length of AC . (2 marks)

(b) In Figure 2, the thin metal sheet ABC is held such that only the vertex B lies on the horizontal ground. D and E are points lying on the horizontal ground vertically below the vertices A and C respectively. AC produced meets the horizontal ground at the point F . A craftsman finds that $AD = 10$ cm and $CE = 2$ cm .

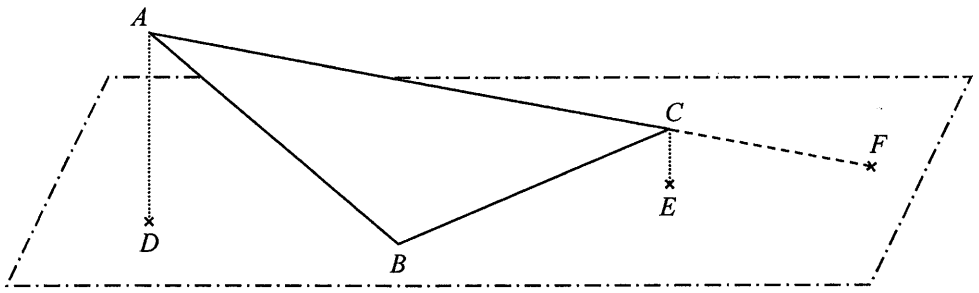

Figure 2

(i) Find the distance between C and F .

(ii) Find the area of ΔABF .

(iii) Find the inclination of the thin metal sheet ABC to the horizontal ground.

(iv) The craftsman claims that the area of ΔBDF is greater than 460 cm^2 . Do you agree? Explain your answer.

(11 marks)

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