Mathematics Compulsory Part Paper 1

Solution	Marks	Remarks
1. $k=\dfrac{3x-y}{y}$ $yk=3x-y$ $y=\dfrac{3x}{k+1}$		
2. $\dfrac{(m^4n^{-1})^3}{(m^{-2})^5}$ $=\dfrac{m^{12}n^{-3}}{m^{-10}}$ $=\dfrac{m^{22}}{n^3}$		
3. (a) $x^2-4xy+3y^2=(x-3y)(x-y)$ (b) $x^2-4xy+3y^2+11x-33y$ $=(x-3y)(x-y)+11(x-3y)$ $=(x-3y+11)(x-y)$		
4. Let x, y be the number of regular tickets and concessionary tickets sold respectively $\begin{cases}x=5y\\126x+78y=50976\end{cases}$ $126x+78\left(\dfrac{x}{5}\right)=50976$ $\begin{cases}x=360\\y=72\end{cases}$ The number of admission tickets sold that day $=432$		
5. (a) $7(x-2)\le\dfrac{11x+8}{3}$ and $6-x<5$ $21(x-2)\le 11x+8$ and $x>1$ $x\le 5$ and $x>1$ $\therefore 1<x\le 5$ (b) 2, 3, 4, 5 are the only integers satisfying (a) So, there are 4 integers satisfying both inequalities in (a)		

Solution	Marks	Remarks

6. (a) The coordinates of $A' = (-4, -3)$
The coordinates of $B' = (9, 9)$

(b) Slope of AB
$= \dfrac{-9-4}{9-(-3)}$
$= -\dfrac{13}{12}$
Slope of A'B'
$= \dfrac{-3-9}{-4-9}$
$= \dfrac{12}{13}$
Slope of AB × Slope of A'B'
$= -\dfrac{13}{12} \times \dfrac{12}{13}$
$= -1$
$\therefore AB \perp A'B'$

7. (a) $\dfrac{x}{360°} = \dfrac{1}{9}$
$x = 40°$

(b) Let N be the number of students in the school
$\dfrac{180}{N} = \dfrac{360° - 90° - 40° - 158°}{360°}$
$N = 900$
The number of students in the school
$= 900$

8. (a) $y = \dfrac{k}{\sqrt{x}}$
When $y = 81$, $x = 144$
$81 = \dfrac{k}{\sqrt{144}}$
$k = 972$
$y = \dfrac{972}{\sqrt{x}}$

(b) Change in value of y
$= \dfrac{972}{\sqrt{324}} - \dfrac{972}{\sqrt{144}}$
$= -27$

Solution	Marks	Remarks
9. (a) Let x mL be the actual capacity of a standard bottle		
$200-\frac{10}{2}\le x<200+\frac{10}{2}$		
$195\le x<205$		
The least possible capacity is 195 mL		
(b) $23400\le 120x<24600$		
$23.4L\le 120x<24.6L$		
Least total capacity $=23.4\ L$		
No, I don't agree the claim.		
10. (a) $\text{OP}=\text{OR}(given)$		
$PS=RS\,(given)$		
$OS=OS\,(common)$		
$\therefore \Delta OPS\cong\Delta ORS\,(SSS)$		
(b) $\angle POQ=10°$		
$\angle POQ=\angle QOR=10°$		
$\angle POR=20°$		
Area of the sector OPQR		
$=\frac{20°}{360°}\times\pi\times 6^2$		
$=2\pi\text{ cm}^2$		
11. (a) $\frac{895+70+a+80+b}{15}=70$		
$a+b=5$		
$80+b-61=22$		
$b=3$		
$a=2$		
median $=\$69$		
standard deviation		
$=\$7.330302404$		
$\approx\$7.33$ (corr to 3 sig fig)		

Solution	Marks	Remarks
12. (a) Let V_1 cm^3 and V_2 cm^3 be the volume of smaller right pyramid and larger right pyramid respectively $\frac{V_1}{V_2}$ $=\left(\sqrt{\frac{4}{9}}\right)^3$ $=\frac{8}{27}$ V_1+V_2 $=(84)(20)$ $=1680$ V_2 $=1680\times\frac{27}{27+8}$ $=1296$ The volume of the larger pyramid $=1296\text{ cm}^3$		
(b) Volume of smaller pyramid $=384\text{ cm}^3$ Height of smaller pyramid $=\frac{2}{3}\times 12$ $=8$ $\frac{1}{3}(\text{Base Area})(8)=384$ Base Area $=144\text{cm}^2$ Length of square in base $=\sqrt{144}$ $=12$ Total surface area $=\frac{1}{2}(12)\sqrt{\left(\frac{12}{2}\right)^2+8^2}\times 4+144$ $=384\text{ cm}^2$		

Solution			Marks	Remarks
13.	(a)	Let the equation of C be $(x-4)^2+(y+1)^2=r$, where r is a real constant Since C passes through $(-6,5)$, So, we have $(-6-2)^2+(5+1)^2=r$ $r=100$ The equation of C: $(x-4)^2+(y+1)^2=100$		
	(b)	Radius of $C=10$ GF $=\sqrt{(2-(-3))^2+(-1-11)^2}$ $=13$ >10 F lies outside C		
	(c)	(i) F, G, H are collinear (ii) The equation of straight line which passes through F and H: $\frac{y-11}{x+3}=\frac{11-(-1)}{-3-2}$ $y=-\frac{12}{5}x+\frac{19}{5}$ $12x+5y-19=0$		

Solution	Marks	Remarks
14. (a) $\mathrm{f}(x)$ $=(3x+7)(2x^2+ax+4)+bx+c$ $=6x^3+(3a+14)x^2+(12+7a+b)x+28+c$ $\equiv 6x^3-13x^2-46x+34$ $3a+14=-13$ $a=-9$		
(b) (i) Since $\mathrm{g}(x)$ is a quadratic polynomial, degree of quotient when $\mathrm{g}(x)$ is divided by $2x^2+ax+4$ is 0 $\mathrm{g}(x)=A(2x^2-9x+4)+bx+c$, where A is a constant $\mathrm{f}(x)-\mathrm{g}(x)$ $=(3x+7)(2x^2+ax+4)+bx+c-(A(2x^2-9x+4)+bx+c)$ $=(3x+7-A)(2x^2+ax+4)$ $\therefore$ $\mathrm{f}(x)-\mathrm{g}(x)$ is divisible by $2x^2+ax+4$		
(b) (ii) $\mathrm{f}(x)-\mathrm{g}(x)=0$ $(3x+7-A)(2x^2-9x+4)=0$ $x=\frac{A-7}{3}$ or $x=4$ or $x=\frac{1}{2}$ Since $\frac{1}{2}$ is not an integer, I don't agree the claim		

Solution	Marks	Remarks
15. $\begin{cases} 0 = a + \log_b 9 \\ 3 = a + \log_b 243 \end{cases}$ $3 = \log_b 243 - \log_b 9$ $3 = \log_b \frac{243}{9}$ $b^3 = 27$ $b = 3$ $a = -2$ $y = -2 + \log_3 x$ $y + 2 = \log_3 x$ $x = 3^{y+2}$		
16. (a) The total volume of water imported $= 1.5\times10^7 + 0.9\times1.5\times10^7 + 0.9^2\times1.5\times10^7 + ... + 0.9^{19}\times1.5\times10^7$ $= 1.5\times10^7\times(1 + 0.9 + 0.9^2 + ... + 0.9^{19})$ $= 1.5\times10^7\times\frac{1-0.9^{20}}{1-0.9}$ $= 1.31763501\times10^8$ $\approx 1.32\times10^8\ \text{m}^3$ (corr to 3 sig fig)		
(b) Since the water imported every year is positive, The total volume of water imported $< 1.5\times10^7 + 0.9\times1.5\times10^7 + 0.9^2\times1.5\times10^7 + ...$ $= 1.5\times10^7\times(1 + 0.9 + 0.9^2 + ...)$ $= 1.5\times10^7\times\frac{1}{1-0.9}$ $= 1.5\times10^8$ $< 1.6\times10^8$ No, I don't agree the claim.		
Suppose the total water imported can exceed $1.6\times10^8\ \text{m}^3$ at n^{th} year $1.5\times10^7 + 0.9\times1.5\times10^7 + 0.9^2\times1.5\times10^7 + ... + 0.9^{n-1}\times1.5\times10^7 > 1.6\times10^8$ $1.5\times10^7\times(1 + 0.9 + 0.9^2 + ... + 0.9^{n-1}) > 1.6\times10^8$ $\frac{1-0.9^n}{1-0.9} > \frac{32}{3}$ $0.9^n < -\frac{1}{15}$ which is impossible No, I don't agree the claim.		

Solution	Marks	Remarks
17. (a) Required probability		
$=\dfrac{C_4^4 C_1^{15}}{C_5^{19}}$		
$=\left(\dfrac{4}{19}\right)\left(\dfrac{3}{18}\right)\left(\dfrac{2}{17}\right)\left(\dfrac{1}{16}\right)\left(\dfrac{15}{15}\right)C_4^5$		
$=\dfrac{5}{3876}$		r.t. 0.00129
(b) Required probability		
$=\dfrac{C_3^4 C_2^{15}}{C_5^{19}}$		
$=\left(\dfrac{4}{19}\right)\left(\dfrac{3}{18}\right)\left(\dfrac{2}{17}\right)\left(\dfrac{15}{16}\right)\left(\dfrac{14}{15}\right)C_3^5$		
$=\dfrac{35}{969}$		r.t. 0.0361
(c) Required probability		
$=1-\dfrac{5}{3876}-\dfrac{35}{969}$		
$=\left(\dfrac{15}{19}\right)\left(\dfrac{14}{18}\right)\left(\dfrac{13}{17}\right)\left(\dfrac{12}{16}\right)\left(\dfrac{11}{15}\right)C_0^5+\left(\dfrac{4}{19}\right)\left(\dfrac{15}{18}\right)\left(\dfrac{14}{17}\right)\left(\dfrac{13}{16}\right)\left(\dfrac{12}{15}\right)C_1^5$ $+\left(\dfrac{4}{19}\right)\left(\dfrac{3}{18}\right)\left(\dfrac{15}{17}\right)\left(\dfrac{14}{16}\right)\left(\dfrac{13}{15}\right)C_2^5$		
$=\dfrac{3731}{3876}$		r.t. 0.963

Solution	Marks	Remarks
18. (a) $\begin{cases} y=19 \\ y=2x^2-2kx+2x-3k+8 \end{cases}$ $19=2x^2-2kx+2x-3k+8$ $2x^2-2kx+2x-3k-11=0$ Δ $=(-2k+2)^2-4(2)(-3k-11)$ $=4k^2+16k+92$ $=4(k+2)^2+76$ ≥ 76 , for all real values of k $\therefore \Delta > 0$, for all real values of k L and Γ intersect at two distinct points.		
(b) (i) a, b are the roots of $2x^2-2kx+2x-3k-11=0$ ab $=\frac{-3k-11}{2}$ $=-\frac{3k+11}{2}$ $a+b$ $=-\frac{-2k+2}{2}$ $=k-1$ $(a-b)^2$ $=a^2+b^2-2ab$ $=(a+b)^2-4ab$ $=(k-1)^2-4(-\frac{3k+11}{2})$ $=k^2-2k+1+6k+22$ $=k^2+4k+23$		
(ii) AB $=\sqrt{(a-b)^2}$ $=\sqrt{k^2+4k+23}$ $=\sqrt{(k+2)^2+19}$ $\geq \sqrt{19}=4.358898944$, for all real values of k No, it is impossible.		

Solution	Marks	Remarks
19. (a) $\dfrac{AC}{\sin(180^\circ - 30^\circ - 42^\circ)} = \dfrac{24}{\sin 30^\circ}$ $AC = 45.65071278$ $AC \approx 45.7\text{ cm}$ (corr to 3 sig fig) The length of AC is 45.7 cm		
(b) (i) $\dfrac{CF}{CF + AC} = \dfrac{2}{10}$ $CF = \dfrac{1}{4}AC$ $CF = \dfrac{1}{4} \times 45.65071278$ $CF = 11.4126782$ $CF \approx 11.4\text{ cm}$ (corr to 3 sig fig)		
(ii) Area of $\Delta ABF = \dfrac{1}{2}(AB)(AF)\sin 30^\circ$ $AF = CF + AC = 57.06339098$ $AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos 42^\circ$ $AB^2 = (45.65071278)^2 + 24^2 - 2(24)(45.65071278)\cos 42^\circ$ $AB = 32.11826911$ Area of ΔABF $= \dfrac{1}{2}(32.11826911)(57.06339098)\sin 30^\circ$ $= 458.1943369\text{ cm}^2$ $\approx 458\text{ cm}^2$ (corr to 3 sig fig)		

Solution	Marks	Remarks
(iii) $BF^2 = AF^2 + AB^2 - 2(AF)(AB)\cos 30°$ $BF^2 = (57.06339098)^2 + (32.11826911)^2 - 2(57.06339098)(32.11826911)\cos 30°$ $BF = 33.36690449$ Let h be the height of ΔABF with base BF $\frac{1}{2}(h)(BF) =$ Area of ΔABF $\frac{1}{2}(h)(BF) = 458.1943369$ $\frac{1}{2}(h)(33.36690449) = 458.1943369$ $h = 27.46400026$ Let the inclination of the thin metal sheet ABC to horizontal ground be θ $\sin\theta = \frac{10}{h}$ $\sin\theta = \frac{10}{27.46400026}$ $\theta = 21.35300646°$ The inclination of the thin metal sheet ABC to horizontal ground = $21.4°$		
(iv) $BF = 33.36690449$ $BD = \sqrt{AB^2 - 10^2}$ $BD = 30.52184808$ $DF = \sqrt{CF^2 - 10^2}$ $DF = 56.18033989$ Let $s = \frac{BF + BD + DF}{2} = 60.03454623$ Area of ΔBDF $= \sqrt{s(s-BF)(s-BD)(s-DF)}$ $= 426.741482$ < 460 No, I don't agree the claim.		

Paper 2

Question No.	Key	Question No.	Key
1.	A	26.	A
2.	D	27.	B
3.	A	28.	C
4.	D	29.	B
5.	D	30.	B
6.	A	31.	D
7.	B	32.	D
8.	A	33.	C
9.	C	34.	D
10.	C	35.	B
11.	B	36.	C
12.	C	37.	C
13.	B	38.	A
14.	B	39.	A
15.	C	40.	B
16.	D	41.	D
17.	D	42.	B
18.	A	43.	C
19.	D	44.	B
20.	D	45.	A
21.	C		
22.	D		
23.	A		
24.	A		
25.	C		

Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.