INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the ‘Time is up’ announcement.
SECTION A(1) (35 marks)

1. Simplify \( \left( \frac{m^5 n^{-2}}{m^2 n^{-3}} \right)^6 \) and express your answer with positive indices. (3 marks)

2. Make \( a \) the subject of the formula \( \frac{5 + b}{1 - a} = 3b \). (3 marks)
3. Factorize
   (a) \(9x^2 - 42xy + 49y^2\),
   (b) \(9x^2 - 42xy + 49y^2 - 6x + 14y\). (3 marks)

4. The cost of a chair is \$360. If the chair is sold at a discount of 20\% on its marked price, then the percentage profit is 30\%. Find the marked price of the chair. (4 marks)
5. The ratio of the capacity of a bottle to that of a cup is $4:3$. The total capacity of 7 bottles and 9 cups is 11 litres. Find the capacity of a bottle. (4 marks)

6. In a polar coordinate system, the polar coordinates of the points $A$, $B$ and $C$ are $(13, 157^\circ)$, $(14, 247^\circ)$ and $(15, 337^\circ)$ respectively.

(a) Let $O$ be the pole. Are $A$, $O$ and $C$ collinear? Explain your answer.

(b) Find the area of $\triangle ABC$. (4 marks)
7. In Figure 1, $BD$ is a diameter of the circle $ABCD$. If $AB = AC$ and $\angle BDC = 36^\circ$, find $\angle ABD$.

(4 marks)
8. The coordinates of the points $A$ and $B$ are $(-3, 4)$ and $(-2, -5)$ respectively. $A'$ is the reflection image of $A$ with respect to the $y$-axis. $B$ is rotated anticlockwise about the origin $O$ through $90^\circ$ to $B'$.

(a) Write down the coordinates of $A'$ and $B'$.

(b) Let $P$ be a moving point in the rectangular coordinate plane such that $P$ is equidistant from $A'$ and $B'$. Find the equation of the locus of $P$.

(5 marks)
The following table shows the distribution of the numbers of online hours spent by a group of children on a certain day.

<table>
<thead>
<tr>
<th>Number of online hours</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of children</td>
<td>( r )</td>
<td>8</td>
<td>12</td>
<td>( s )</td>
</tr>
</tbody>
</table>

It is given that \( r \) and \( s \) are positive numbers.

(a) Find the least possible value and the greatest possible value of the inter-quartile range of the distribution.

(b) If \( r = 9 \) and the median of the distribution is 3, how many possible values of \( s \) are there? Explain your answer.

(5 marks)
SECTION A(2) (35 marks)

10. Let \( f(x) \) be a polynomial. When \( f(x) \) is divided by \( x - 1 \), the quotient is \( 6x^2 + 17x - 2 \). It is given that \( f(1) = 4 \).

(a) Find \( f(-3) \).  

(b) Factorize \( f(x) \).
11. Let $C$ be the cost of manufacturing a cubical carton of side $x$ cm. It is given that $C$ is partly constant and partly varies as the square of $x$. When $x = 20$, $C = 42$; when $x = 120$, $C = 112$.

(a) Find the cost of manufacturing a cubical carton of side 50 cm. (4 marks)

(b) If the cost of manufacturing a cubical carton is $58$, find the length of a side of the carton. (2 marks)
12. Figure 2 shows the graphs for Ada and Billy running on the same straight road between town $P$ and town $Q$ during the period 1:00 to 3:00 in an afternoon. Ada runs at a constant speed. It is given that town $P$ and town $Q$ are 16 km apart.

(a) How long does Billy rest during the period? (2 marks)

(b) How far from town $P$ do Ada and Billy meet during the period? (3 marks)

(c) Use average speed during the period to determine who runs faster. Explain your answer. (2 marks)
13. The bar chart below shows the distribution of the most favourite fruits of the students in a group. It is given that each student has only one most favourite fruit.

If a student is randomly selected from the group, then the probability that the most favourite fruit is apple is \( \frac{3}{20} \).

(a) Find \( k \). (3 marks)

(b) Suppose that the above distribution is represented by a pie chart.
   
   (i) Find the angle of the sector representing that the most favourite fruit is orange.

   (ii) Some new students now join the group and the most favourite fruit of each of these students is orange. Will the angle of the sector representing that the most favourite fruit is orange be doubled? Explain your answer. (4 marks)
14. In Figure 3, \( OABC \) is a circle. It is given that \( AB \) produced and \( OC \) produced meet at \( D \).

(a) Write down a pair of similar triangles in Figure 3.  

(b) Suppose that \( \angle AOD = 90^\circ \). A rectangular coordinate system, with \( O \) as the origin, is introduced in Figure 3 so that the coordinates of \( A \) and \( D \) are \((6,0)\) and \((0,12)\) respectively. If the ratio of the area of \( \triangle BCD \) to the area of \( \triangle OAD \) is \( 45:16 \), find

(i) the coordinates of \( C \),

(ii) the equation of the circle \( OABC \).
15. The mean score of a class of students in a test is 48 marks. The scores of Mary and John in the test are 36 marks and 66 marks respectively. The standard score of Mary in the test is $-2$.

(a) Find the standard score of John in the test. (2 marks)

(b) A student, David, withdraws from the class and his test score is then deleted. It is given that his test score is 48 marks. Will there be any change in the standard score of John due to the deletion of the test score of David? Explain your answer. (2 marks)
16. There are 18 boys and 12 girls in a class. From the class, 4 students are randomly selected to form the class committee.

(a) Find the probability that the class committee consists of boys only. (2 marks)

(b) Find the probability that the class committee consists of at least 1 boy and 1 girl. (2 marks)
17. (a) Express \( \frac{1}{1+2i} \) in the form of \( a+bi \), where \( a \) and \( b \) are real numbers. (2 marks)

(b) The roots of the quadratic equation \( x^2 + px + q = 0 \) are \( \frac{10}{1+2i} \) and \( \frac{10}{1-2i} \). Find

(i) \( p \) and \( q \),

(ii) the range of values of \( r \) such that the quadratic equation \( x^2 + px + q = r \) has real roots. (5 marks)
18. Figure 4 shows a geometric model $ABCD$ in the form of tetrahedron. It is found that $\angle ACB = 60^\circ$, $AC = AD = 20$ cm, $BC = BD = 12$ cm and $CD = 14$ cm.

(a) Find the length of $AB$. (2 marks)

(b) Find the angle between the plane $ABC$ and the plane $ABD$. (4 marks)

(c) Let $P$ be a movable point on the slant edge $AB$. Describe how $\angle CPD$ varies as $P$ moves from $A$ to $B$. Explain your answer. (2 marks)
19. The amount of investment of a commercial firm in the 1st year is $4000000. The amount of investment in each successive year is \( r\% \) less than the previous year. The amount of investment in the 4th year is $1048576.

(a) Find \( r \). (2 marks)

(b) The revenue made by the firm in the 1st year is $2000000. The revenue made in each successive year is 20\% \text{ less than the previous year.}

(i) Find the least number of years needed for the total revenue made by the firm to exceed $9000000.

(ii) Will the total revenue made by the firm exceed $1000000? Explain your answer.

(iii) The manager of the firm claims that the total revenue made by the firm will exceed the total amount of investment. Do you agree? Explain your answer. (10 marks)
INSTRUCTIONS

1. Read carefully the instructions on the Answer Sheet. After the announcement of the start of the examination, you should first stick a barcode label and insert the information required in the spaces provided. No extra time will be given for sticking on the barcode label after the ‘Time is up’ announcement.

2. When told to open this book, you should check that all the questions are there. Look for the words ‘END OF PAPER’ after the last question.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS.** You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.
There are 30 questions in Section A and 15 questions in Section B. The diagrams in this paper are not necessarily drawn to scale. Choose the best answer for each question.

Section A

1. \( x^3(2x + x) = \)
   
   A. \( 3x^4 \)
   
   B. \( 2x^5 \)
   
   C. \( 3x^5 \)
   
   D. \( 2x^6 \)

2. If \( 3a + 1 = 3(b - 2) \), then \( b = \)
   
   A. \( a + 1 \)
   
   B. \( a + 3 \)
   
   C. \( a + \frac{7}{3} \)
   
   D. \( a - \frac{5}{3} \)

3. \( p^2 - q^2 - p - q = \)
   
   A. \( (p + q)(p - q - 1) \)
   
   B. \( (p + q)(p + q - 1) \)
   
   C. \( (p - q)(p - q + 1) \)
   
   D. \( (p - q)(p + q - 1) \)
4. Let \( m \) and \( n \) be constants. If \( m(x - 3)^2 + n(x + 1)^2 = x^2 - 38x + 41 \), then \( m = \)

A. \(-4\).
B. \(-1\).
C. \(3\).
D. \(5\).

5. Let \( f(x) = x^4 - x^3 + x^2 - x + 1 \). When \( f(x) \) is divided by \( x + 2 \), the remainder is

A. \(-2\).
B. \(0\).
C. \(11\).
D. \(31\).

6. Let \( k \) be a constant. If the quadratic equation \( 3x^2 + 2kx - k = 0 \) has equal roots, then \( k = \)

A. \(-3\).
B. \(3\).
C. \(-3\) or \(0\).
D. \(0\) or \(3\).

7. In the figure, the \( x \)-intercepts of the straight lines \( L_1 \) and \( L_2 \) are 5 while the \( y \)-intercepts of the straight lines \( L_2 \) and \( L_3 \) are 3. Which of the following are true?

I. The equation of \( L_1 \) is \( x = 5 \).
II. The slope of \( L_2 \) is \( \frac{3}{5} \).
III. The point \((2, 3)\) lies on \( L_3 \).

A. I and II only
B. I and III only
C. II and III only
D. I, II and III
8. The figure shows the graph of \( y = ax^2 - 2x + b \), where \( a \) and \( b \) are constants. Which of the following is/are true?

I. \( a > 0 \)
II. \( b < 0 \)
III. \( ab < 1 \)

A. I only  
B. II only  
C. I and III only  
D. II and III only

9. The solution of \( 4x > x - 3 \) or \( 3 - x < x + 7 \) is

A. \( x > -2 \)  
B. \( x < -2 \)  
C. \( x > -1 \)  
D. \( x < -2 \) or \( x > -1 \)

10. John buys a vase for \( $1600 \). He then sells the vase to Susan at a profit of \( 20\% \). At what price should Susan sell the vase in order to have a profit of \( 20\% \)?

A. \( $2240 \)  
B. \( $2304 \)  
C. \( $2400 \)  
D. \( $2500 \)

11. If the circumference of a circle is increased by \( 40\% \), then the area of the circle is increased by

A. \( 18\% \)  
B. \( 20\% \)  
C. \( 40\% \)  
D. \( 96\% \)
12. Let $\alpha$ and $\beta$ be non-zero constants. If $(\alpha + \beta):(3\alpha - \beta) = 7:3$, then $\alpha:\beta =$

A. $5:9$.
B. $9:5$.
C. $19:29$.
D. $29:19$.

13. If $z$ varies directly as $x$ and inversely as $y^2$, which of the following must be constant?

A. $\frac{x}{y^2z}$
B. $\frac{z}{xy^2}$
C. $\frac{yz}{x}$
D. $\frac{xz}{y^2}$

14. $0.009049999 =$

A. $0.00905$ (correct to 3 decimal places).
B. $0.00905$ (correct to 3 significant figures).
C. $0.00905$ (correct to 6 decimal places).
D. $0.00905$ (correct to 6 significant figures).

15. In the figure, $O$ is the centre of the sector $OABC$. If the area of $\Delta OAC$ is $12 \text{ cm}^2$, find the area of the segment $ABC$.

A. $3(\pi - 2) \text{ cm}^2$
B. $3(\pi - 1) \text{ cm}^2$
C. $6(\pi - 2) \text{ cm}^2$
D. $6(\pi - 1) \text{ cm}^2$
16. The figure shows a right circular cone of height 8 cm and slant height 17 cm. Find the volume of the circular cone.

A. $255\pi \text{ cm}^3$
B. $345\pi \text{ cm}^3$
C. $480\pi \text{ cm}^3$
D. $600\pi \text{ cm}^3$

17. In the figure, $ABCD$ is a rectangle. $E$ is the mid-point of $BC$. $F$ is a point lying on $CD$ such that $DF = 2CF$. If the area of $\triangle CEF$ is $1 \text{ cm}^2$, then the area of $\triangle AEF$ is

A. $2 \text{ cm}^2$
B. $3 \text{ cm}^2$
C. $4 \text{ cm}^2$
D. $6 \text{ cm}^2$

18. In the figure, $AB = 4 \text{ cm}$, $BC = CD = DE = 8 \text{ cm}$ and $FG = 9 \text{ cm}$. Find the perimeter of $\triangle AEH$.

A. $60 \text{ cm}$
B. $74 \text{ cm}$
C. $150 \text{ cm}$
D. $164 \text{ cm}$
19. In the figure, $AB = BC$ and $D$ is a point lying on $BC$ such that $CD = DE$. If $AB \parallel CE$, find $\angle CDE$.

A. 52°
B. 58°
C. 64°
D. 76°

20. In the figure, $O$ is the centre of the semi-circle $ABCD$. $AC$ and $BD$ intersect at $E$. If $AD \parallel OC$, then $\angle AED =$

A. 48°
B. 55°
C. 57°
D. 66°

21. In the figure, $O$ is the centre of the circle $ABCD$. If $\widehat{AB} = \widehat{BC} = 2\widehat{CD}$, then $\angle BCD =$

A. 64°
B. 87°
C. 93°
D. 116°
22. In the figure, $ABCD$ is a square. $F$ is a point lying on $AD$ such that $CF \parallel BE$. If $AB = AE$, find $\angle ABF$ correct to the nearest degree.

A. 17°
B. 18°
C. 22°
D. 26°

23. For $0^\circ \leq \theta \leq 90^\circ$, the least value of $\frac{30}{3\sin^2 \theta + 2\sin^2 (90^\circ - \theta)}$ is

A. 5.
B. 6.
C. 10.
D. 15.

24. Which of the following parallelograms have rotational symmetry and reflectional symmetry?

I.  
II.  
III.  

A. I and II only
B. I and III only
C. II and III only
D. I, II and III
25. If the point \((-2, -1)\) is reflected with respect to the straight line \(y = -5\), then the coordinates of its image are

A. \((-8, -1)\).
B. \((-2, -9)\).
C. \((-2, 11)\).
D. \((12, -1)\).

26. The coordinates of the points \(A\) and \(B\) are \((1, -3)\) and \((-5, 7)\) respectively. If \(P\) is a point lying on the straight line \(y = x + 2\) such that \(AP = PB\), then the coordinates of \(P\) are

A. \((-2, 0)\).
B. \((-2, 2)\).
C. \((0, 2)\).
D. \((3, 5)\).

27. The equation of a circle is \(2x^2 + 2y^2 + 8x - 12y + 3 = 0\). Which of the following are true?

I. The coordinates of the centre of the circle are \((-2, 3)\).
II. The radius of the circle is 7.
III. The point \((2, 3)\) lies outside the circle.

A. I and II only
B. I and III only
C. II and III only
D. I, II and III
28. Two numbers are randomly drawn at the same time from four cards numbered 2, 3, 5 and 7 respectively. Find the probability that the sum of the numbers drawn is a multiple of 4.

A. \(\frac{1}{3}\)  
B. \(\frac{1}{4}\)  
C. \(\frac{1}{6}\)  
D. \(\frac{5}{16}\)

29. The box-and-whisker diagram below shows the distribution of the heights (in cm) of some students. Which of the following is/are true?

I. The height of the tallest student is 180 cm.
II. The inter-quartile range of the distribution is 15 cm.
III. Less than half of the students are taller than 170 cm.

A. I only  
B. II only  
C. I and III only  
D. II and III only

30. The figure below shows the cumulative frequency polygons of the test score distributions X and Y. Let \(m_1\), \(r_1\) and \(s_1\) be the median, the range and the standard deviation of X respectively while \(m_2\), \(r_2\) and \(s_2\) be the median, the range and the standard deviation of Y respectively. Which of the following are true?

I. \(m_1 > m_2\)
II. \(r_1 > r_2\)
III. \(s_1 > s_2\)

A. I and II only  
B. I and III only  
C. II and III only  
D. I, II and III
Section B

31. The figure above shows the graph of \( y = f(x) \). If \( 2f(x) = g(x) \), which of the following may represent the graph of \( y = g(x) \) ?

A. ![Graph A]
B. ![Graph B]
C. ![Graph C]
D. ![Graph D]

32. B0000000023\textsubscript{16} =

A. \( 11 \times 16^0 + 23 \).
B. \( 11 \times 16^0 + 35 \).
C. \( 12 \times 16^1 + 23 \).
D. \( 12 \times 16^1 + 35 \).
33. If the roots of the quadratic equation $x^2 - kx + 3 = 0$ are $\alpha$ and $\beta$, then $\alpha^3 + \beta^3 =$

A. $k^3$.

B. $k^3 - 3k$.

C. $k^3 - 9k$.

D. $k^3 - 12k$.

34. If $x$ is a real number, then the real part of $(x + 3i)(3 + i)$ is

A. $3x$.

B. $x + 3$.

C. $3x + 3$.

D. $3x - 3$.

35. The $n$th term of a sequence is $2n + 3$. If the sum of the first $m$ terms of the sequence is less than 3000, then the greatest value of $m$ is

A. 52.

B. 53.

C. 56.

D. 57.

36. Let $b > 1$. If $a = \log_{12} b$, then $\frac{1}{a} =$

A. $\log_b \frac{1}{12}$.

B. $\log_b 12$.

C. $\log_{12} \frac{1}{b}$.

D. $\frac{1}{\log_b 12}$.
37. The graph in the figure shows the linear relation between \( \log_3 t \) and \( \log_3 x \). If \( x = kt^n \), then \( k = \)

A. \( \frac{1}{81} \).

B. \( 81 \).

C. \( \frac{-4}{5} \).

D. \( \frac{-5}{4} \).

38. Let \( a \) be a constant and \( -90^\circ < \theta < 90^\circ \). If the figure shows the graph of \( y = a \sin(x^\circ + \theta) \), then

A. \( a = -2 \) and \( \theta = -45^\circ \).

B. \( a = -2 \) and \( \theta = 45^\circ \).

C. \( a = 2 \) and \( \theta = -45^\circ \).

D. \( a = 2 \) and \( \theta = 45^\circ \).

39. The figure shows a right prism \( ABCDEF \) with a right-angled triangle as the cross-section. \( A, B, E \) and \( F \) lie on the horizontal ground. \( G \) is a point lying on \( AB \) such that \( AG:GB = 5:3 \). If \( \angle DAE = a \), \( \angle CBF = b \), \( \angle CGF = c \) and \( \angle DGE = d \), which of the following is true?

A. \( a > c > d \)

B. \( a > d > c \)

C. \( c > b > d \)

D. \( c > d > b \)
40. In the figure, $A$ is the common centre of the two circles. $BC$ is a chord of the larger circle and touches the smaller circle at $D$. $AD$ produced meets the larger circle at $E$. $F$ is a point lying on the smaller circle such that $E$, $D$, $A$ and $F$ are collinear. If $BC = 24$ cm and $DE = 8$ cm, then $EF =$

A. 13 cm
B. 16 cm
C. 18 cm
D. 20 cm

41. If the straight line $x - y = 0$ and the circle $x^2 + y^2 + 6x + ky - k = 0$ do not intersect with each other, find the range of values of $k$.

A. $2 < k < 18$
B. $-18 < k < -2$
C. $k < 2$ or $k > 18$
D. $k < -18$ or $k > -2$

42. Let $O$ be the origin. If the coordinates of the points $A$ and $B$ are $(18, -24)$ and $(18, 24)$ respectively, then the $x$-coordinate of the orthocentre of $\triangle OAB$ is

A. $-14$
B. $10$
C. $12$
D. $25$
43. Mary, Tom and 8 other students participate in a solo singing contest. If each participant performs once only and the order of performance is randomly arranged, find the probability that Mary performs just after Tom.

A. \( \frac{1}{2} \)

B. \( \frac{1}{10} \)

C. \( \frac{1}{45} \)

D. \( \frac{1}{90} \)

44. The mean, the variance and the inter-quartile range of a set of numbers are 40, 9 and 18 respectively. If 5 is added to each number of the set and each resulting number is then tripled to form a new set of numbers, find the mean, the variance and the inter-quartile range of the new set of numbers.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Variance</th>
<th>Inter-quartile range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 120</td>
<td>27</td>
<td>69</td>
</tr>
<tr>
<td>B. 120</td>
<td>81</td>
<td>69</td>
</tr>
<tr>
<td>C. 135</td>
<td>27</td>
<td>54</td>
</tr>
<tr>
<td>D. 135</td>
<td>81</td>
<td>54</td>
</tr>
</tbody>
</table>

45. Let \( A \) be a group of numbers \( \{ \alpha, \beta, \gamma, \delta \} \) and \( B \) be another group of numbers \( \{ \alpha + 2, \beta + 2, \mu + 2, \gamma + 2, \delta + 2 \} \), where \( \alpha < \beta < \mu < \gamma < \delta \). Which of the following must be true?

I. The median of \( A \) is smaller than that of \( B \).
II. The range of \( A \) and the range of \( B \) are the same.
III. The standard deviation of \( A \) is greater than that of \( B \).

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

END OF PAPER