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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

# PRACTICE PAPER

# MATHEMATICS Extended Part
# Module 1 (Calculus and Statistics)

## Question-Answer Book

(2½ hours)
This paper must be answered in English

**INSTRUCTIONS**

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5 and 7.

2. This paper consists of Section A and Section B.

3. Answer **ALL** questions in Section A. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Answer **ALL** questions in Section B. Write your answers in the other answer book. Start each question (not part of a question) on a new page.

5. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** the book.

6. The Question-Answer book and the answer book will be collected separately at the end of the examination.

7. Unless otherwise specified, all working must be clearly shown.

8. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

9. The diagrams in this paper are not necessarily drawn to scale.

10. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.





*A031E01A*

**Section A** (50 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. (a) Expand $(2x+1)^3$ .

   (b) Expand $e^{-ax}$ in ascending powers of $x$ as far as the term in $x^2$ , where $a$ is a constant.

   (c) If the coefficient of $x^2$ in the expansion of $\frac{(2x+1)^3}{e^{ax}}$ is $-4$ , find the value(s) of $a$ .

(5 marks)

2. It is given that $t = y^3 + 2y^{\frac{-1}{2}} + 1$ and $e^t = x^{x^2+1}$ .

   (a) Find $\frac{\mathrm{d}t}{\mathrm{d}y}$ .

   (b) By expressing $t$ in terms of $x$ , find $\frac{\mathrm{d}t}{\mathrm{d}x}$ .

   (c) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$ .

(5 marks)

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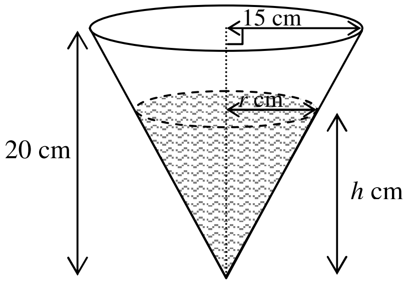


**Figure 1**

A glass container is in the shape of a vertically inverted right circular cone of base radius 15 cm and height 20 cm . Initially, the container is full of water. Suppose the water is running out from it at a constant rate of $2\pi \text{ cm}^3/\text{s}$ . Let $h$ cm be the depth of water remaining in the container, $r$ cm be the radius of the water surface (see Figure 1), $V \text{ cm}^3$ be the volume of the water, and $A \text{ cm}^2$ be the area of the wet surface of the container. It is given that $V = \frac{1}{3}\pi r^2 h$ and $A = \pi r \sqrt{r^2 + h^2}$ .

(a) Express $V$ and $A$ in terms of $r$ only.

(b) When $r = 3$ ,

(i) find the rate of change of the radius of the water surface;

(ii) find the rate of change of the area of the wet surface of the container.

(6 marks)

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4. Consider the curve $C$ : $y = x(2x-1)^{\frac{1}{2}}$ , where $x > \frac{1}{2}$ .

(a) Find $\frac{dy}{dx}$ .

(b) Using (a), find the equations of the two tangents to the curve $C$ which are parallel to the straight line $2x - y = 0$ .

(6 marks)

5. Consider the two curves $C_1 : y = 1 - \frac{e}{e^x}$ and $C_2 : y = e^x - e$ .

(a) Find the $x$-coordinates of all the points of intersection of $C_1$ and $C_2$ .

(b) Find the area of the region bounded by $C_1$ and $C_2$ .

(5 marks)

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6. A random sample of size 10 is drawn from a normal population with mean $\mu$ and variance 8 . Let $\overline{X}$ be the mean of the sample.

(a) Calculate $\operatorname{Var}(2\overline{X}+7)$ .

(b) Suppose the mean of the sample is 50 . Construct a 97% confidence interval for $\mu$ .

(5 marks)

7. In a game, there are two bags, $A$ and $B$ , each containing 5 balls. Bag $A$ contains 3 red and 2 blue balls, while bag $B$ contains 4 red and 1 blue balls. A player first chooses a bag at random and then draws a ball randomly from the bag. The player will be rewarded if the ball drawn is blue. The ball is then replaced for the next player's turn.

(a) Find the probability that a player is rewarded in a particular game.

(b) Two players participate in the game. Given that at least one of them is rewarded, find the probability that both of them are rewarded.

(c) If 60 players are rewarded, find the expected number of players among them having drawn a blue ball from bag $A$ .

(5 marks)



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8. Eggs from a farm are packed in boxes of 30 . The probability that a randomly selected egg is rotten is 0.04 .

(a) Find the probability that a box contains more than 1 rotten egg.

(b) Boxes of eggs are inspected one by one.

(i) Find the probability that the $1^{st}$ box containing more than 1 rotten egg is the $6^{th}$ box inspected.

(ii) What is the expected number of boxes inspected until a box containing more than 1 rotten egg is found?

(7 marks)

9. Suppose $A$ and $B$ are two events. Let $A'$ and $B'$ be the complementary events of $A$ and $B$ respectively. It is given that $P(A \mid B') = 0.6$ , $P(A \cap B) = 0.12$ and $P(A \cap B') = k$ , where $k > 0$ .

(a) Find $P(A)$ , $P(B)$ and $P(A \cup B)$ in terms of $k$ .

(b) If $A$ and $B$ are independent, find the value of $k$ .

(6 marks)

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**Section B** (50 marks)
Answer **ALL** questions in this section and write your answers in the other answer book.

10. An engineer models the rates of the production of an alloy in the first 10 weeks by two new machines $A$ and $B$ respectively by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{61t}{(t+1)^{\frac{5}{2}}} \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{15\ln(t^2+100)}{16} \qquad \text{for } 0 \le t \le 10 \ ,$$

where $x$ (in million kg) and $y$ (in million kg) are the amount of the alloy produced by machines $A$ and $B$ respectively, and $t$ (in weeks) is the time elapsed since the beginning of the production.

(a) Using the substitution $u = t+1$ , find the amount of the alloy produced by machine $A$ in the first 10 weeks. (4 marks)

(b) Using the trapezoidal rule with 5 sub-intervals, estimate the amount of the alloy produced by machine $B$ in the first 10 weeks. (2 marks)

(c) The engineer uses the results of (a) and (b) to claim that machine $B$ is more productive than machine $A$ in the first 10 weeks. Do you agree? Explain your answer. (4 marks)

11. A researcher models the rate of change of the population size of a kind of insects in a forest by

$$\mathrm{P}'(t) = kte^{\frac{a}{20}t} \ ,$$

where $\mathrm{P}(t)$ , in thousands, is the population size, $t$ $(\ge 0)$ is the time measured in weeks since the start of the research, and $a$ , $k$ are integers.
The following table shows some values of $t$ and $\mathrm{P}'(t)$ .

| $t$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\mathrm{P}'(t)$ | 22.83 | 43.43 | 61.97 | 78.60 |

(a) Express $\ln\frac{\mathrm{P}'(t)}{t}$ as a linear function of $t$ . (1 mark)

(b) By plotting a suitable straight line on the graph paper on page 13, estimate the integers $a$ and $k$ . (5 marks)

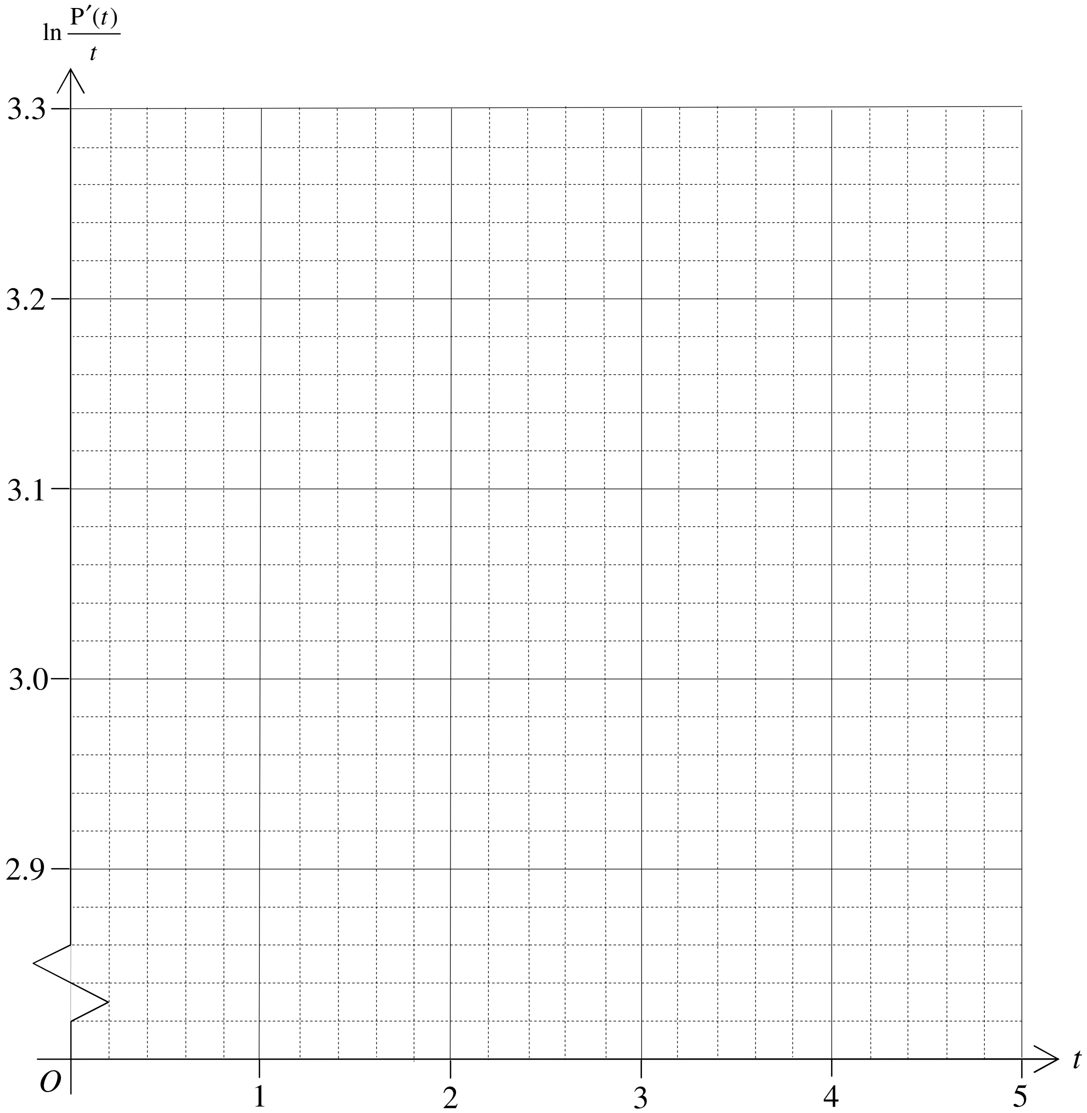
(c) Suppose that $\mathrm{P}(0) = 30$ . Using the estimates in (b),

(i) find the value of $t$ such that the rate of change of the population size of the insect is the greatest;

(ii) find $\frac{\mathrm{d}}{\mathrm{d}t}\left(te^{\frac{a}{20}t}\right)$ and hence, or otherwise, find $\mathrm{P}(t)$ ;

(iii) estimate the population size after a very long time.
[Hint: You may use the fact that $\lim_{t\to\infty}\frac{t}{e^{mt}} = 0$ for any positive constant $m$ .]

(9 marks)



$\ln \frac{P'(t)}{t}$
3.3
3.2
3.1
3.0
2.9
O
1
2
3
4
5
t

12. A staff of a school studies the school sick room utilization. The number of visits to the sick room per day on 100 randomly selected school days are recorded as follows:

| Number of visits per day | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 6 | 12 | 18 | 21 | 20 | 12 | 7 | 4 |

(a) Find an unbiased estimate of the mean number of visits per day.

(1 mark)

(b) (i) Find the sample proportion of school days with less than 4 visits per day.

(ii) Construct an approximate 95% confidence interval for the proportion of school days with less than 4 visits per day.

(3 marks)

(c) Suppose the number of visits per day follows a Poisson distribution with mean $\lambda$ . Assume that the unbiased estimate obtained in (a) is used for $\lambda$ . The sick room is said to be *crowded* on a particular day if there are more than 3 visits on that day.

(i) Find the probability that the sick room is *crowded* on a particular day.

(ii) In a certain week of 5 school days, given that the sick room is *crowded* on at least 2 days, find the probability that the sick room is *crowded* on alternate days in the week.

(6 marks)

13. In a supermarket, there are two cashier counters: a regular counter and an express counter. It is known that 88% of customers pay at the regular counter. It is found that the waiting time for a customer to pay at the regular counter follows the normal distribution with mean 6.6 minutes and standard deviation 1.2 minutes.

(a) Find the probability that the waiting time for a customer to pay at the regular counter is more than 6 minutes.

(2 marks)

(b) Suppose 12 customers who pay at the regular counter are randomly selected. Assume that their waiting times are independent.

(i) Find the probability that there are more than 10 of the 12 customers each having a waiting time of more than 6 minutes.

(ii) Find the probability that the average waiting time of the 12 customers is more than 6 minutes.

(5 marks)

(c) It is found that the waiting time for a customer to pay at the express counter follows the normal distribution with mean $\mu$ minutes and standard deviation 0.8 minutes. It is known that exactly 21.19% of the customers at the regular counter wait less than $k$ minutes, while exactly 3.59% of the customers at the express counter wait more than $k$ minutes.

(i) Find $k$ and $\mu$ .

(ii) Two customers are randomly selected. Assume that their waiting times are independent. Given that both of them wait more than $\mu$ minutes to pay, find the probability that exactly one of them pays at the regular counter.

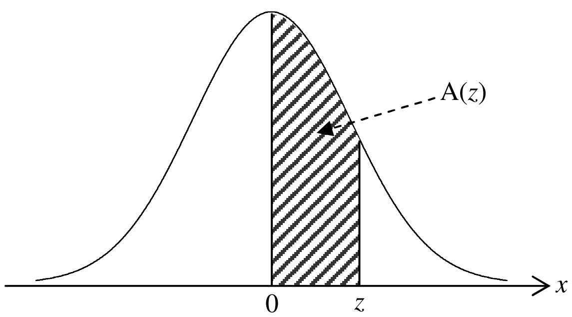
(8 marks)

**END OF PAPER**

## Standard Normal Distribution Table

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |
| 3.5 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 |

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of $z$ can be obtained by symmetry.




$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \, dx$$

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**Answers written on this page will not be marked.**